Quantum Field Theory
Exercises 8

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Loop integrals

1. Show that for d-dimensional vector k

$$\partial_\mu k^\mu = d. \tag{1}$$

2. In d dimensions, the following total derivative vanishes

$$\int d^d k \frac{\partial}{\partial k^\mu} \left(\frac{q^\mu}{P_1^{a_1} \cdots P_N^{a_N}} \right) = 0\,, \tag{2}$$

where P_i are propagators and q^μ can be substituted with any internal or external momentum. Prove the above relation.

Hint:

(a) When q^μ is external, a loop integral exhibits the symmetry

$$I \equiv \int d^d k f(k) = \int d^d k f(k + \lambda\, q)\,. \tag{3}$$

But I must by λ-independent

$$\frac{dI}{d\lambda} = 0\,. \tag{4}$$

(b) The case $q^\mu = k^\mu$, *e.g.* internal, can be proven by using the fact that

$$\int d^d k f(k) = \lambda^d \int d^d k f(\lambda\, k)\,. \tag{5}$$

and this integral, similarly, must be λ-independent, hence its derivative has to vanish.