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QUANTUM FIELD THEORY

EXERCISES 4

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4 Cross-sections

1. *Total (leading-order) cross-section for $\phi\phi \to \phi\phi$*
   Compute the differential cross section for the process $\phi(p_A)\phi(p_B) \to \phi(p_1)\phi(p_2)$ in ϕ^4 scalar theory at $\mathcal{O}(\lambda^2)$ in perturbation theory. Start from the formula for the differential cross section

$$\mathrm{d}\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|} \left(\prod_{f=1}^{2} \frac{\mathrm{d}^3 p_f}{(2\pi)^3} \frac{1}{E_f} \right) (2\pi)^4 \delta^{(4)}(p_A + p_B - p_1 - p_2) |\mathcal{M}(p_A, p_B \to p_1 p_2)|^2$$

   and

   (a) show that the final state phase space can written as

$$\mathrm{d}\Pi_2 \equiv \left(\prod_{f=1}^{2} \frac{\mathrm{d}^3 p_f}{(2\pi)^3} \frac{1}{E_f} \right) (2\pi)^4 \delta^{(4)}(p_A + p_B - p_1 - p_2) = \mathrm{d}\cos\Theta \frac{1}{16\pi} \frac{2|\vec{p_1}|}{E_{cm}^2}$$

   where E_{cm} and Θ are the energy and the angle with respect to the scattering axis in the center-of-mass frame respectively.

   (b) show that the matrix element reduces to

$$|\mathcal{M}(p_A, p_B \to p_1 p_2)|^2 = \lambda^2$$

   (c) show that the total cross section is

$$\sigma = \frac{\lambda^2}{32\pi E_{cm}^2}$$