

IFJ PAN

Theory Division – Particle Theory

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QUANTUM FIELD THEORY

EXERCISES 3

3 Wick's theorem

1. Normal ordering

- (a) Write the following product of field operators in terms of normal ordered and contracted products of field operators using Wick's theorem

$$T \left\{ \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \right\} \quad (3.1)$$

2. Feynman graphs

- (a) Let's assume scalar field theory with ϕ^4 interactions, i.e. the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4. \quad (3.2)$$

Draw all "Feynman"-diagrams (in position space representation) contributing to 4-point correlation function (that includes all connected and disconnected contributions that have no vacuum component):

$$\langle \Omega | T \left\{ \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \right\} | \Omega \rangle \quad (3.3)$$

at λ^0 , λ^1 , λ^2 and λ^3 in perturbation theory.

Hint: Besides using the intuition of what diagrams can contribute you can start from the Gellmann-Low formula

$$\langle \Omega | \phi(x_1) \dots \phi(x_n) | \Omega \rangle = \left. \langle 0 | T \left\{ \phi(x_1) \dots \phi(x_n) \exp \left(-i \int d^4z : \mathcal{H}_{\text{int}}(z) : \right) \right\} | 0 \rangle \right|_{\text{connected graphs}} \quad (3.4)$$

and expand in powers of λ . Rewrite, using Wick's theorem, the obtained time-ordered expression in terms of fully contracted contributions.

(b) Write down the numerical prefactor for each of the diagrams in a).