Lessons from Maschine Learning of Higgs CP measurements in $H \to \tau^+\tau^-, \tau^\pm \to (3\pi)^\pm\nu$, for the `TauSpinner` and 'truth' simulation of leptons.

Z. Was*,

*Institute of Nuclear Physics, Polish Academy of Sciences Krakow

- **(1)** For me all started with 2001 publication on CP observability for $H \to \tau^+\tau^-$, $\tau^\pm \to \pi^\pm\pi^0\nu$. Q: are there similar for other τ decay modes? **A: not much progress despite several attempts.**
- **(2)** In the last two years, we could finally say **yes** for the $\tau^\pm \to (3\pi)^\pm\nu$.
- **(3)** Use of the Maschine Learning algorithm was instrumental to manage observable of 7 dimensional nature (at least). Our choice of particular software, Y. LeCun, Y. Bengio, and G. Hinton, Nature 521 (2015), no. 7553 436 was because of advice/help we could get.
- **(4)** My perspective was what was needed from theoretical (experimental side– direction which need to be worked out in a great detail). Weighted events helpful.
- **(5)** I will present some results, but concentrate on the possible lesson for future applications **and for the design of simulation tools too.**

This work is partly supported by the grant of Narodowe Centrum Nauki 2016/23/B/ST2/03927

Priorities:

- Theory first
- Experimental effects first
- Mathematics first*
- Software development first*

Everybody can choose his own
as *the first*,

but it is natural that the others will do the same.

* See for example https://indico.cern.ch/event/673350/ https://indico.cern.ch/event/687788/

General idea

Only **transverse** spin correlations of τ^+ and τ^- in $H \to \tau^+\tau^-$ are **different for scalar** and pseudoscalar Higgs.

- The correlations can not be measured directly
- One need to measure distributions of τ decay products
- Precisely their transverse (to τ direction in Higgs boson rest frame) momenta
- Easiest to interpret is $\tau^\pm \to \pi^\pm\nu$, but it requre indirect measurement of ν_τ.

M. Kramer, J. H. Kuhn, M. L. Stong and P. M. Zerwas, "Prospects of measuring the parity of Higgs particles," Z. Phys. C **64**, 21 (1994)

A. Rouge, "CP violation in a light Higgs boson decay from tau-spin correlations at a linear collider," Phys. Lett. B **619**, 43 (2005)

- The largest branching ratio (25 %) has $\tau^\pm \to \pi^\pm\pi^0\nu$ and we can look on transverse spin correlations of $\rho^\pm \to \pi^\pm\pi^0$ decays.

There observable based on visible τ decay products only, could be designed.

G. R. Bower, T. Pierzchala, Z. Was and M. Worek, "Measuring the Higgs boson's parity using tau —> rho nu," Phys. Lett. B **543** (2002) 227, K. Desch, ZW, M. Worek, "Measuring the Higgs boson parity at a linear collider using the tau impact parameter and tau $\to$ rho nu decay", Eur.Phys.J. C29 (2003) 491

Important: no dependence on production process

The Higgs boson's parity

- H/A parity information can be extracted from the correlations between τ^+ and τ^- spin components which are further reflected in correlations between the τ decay products in the plane transverse to the $\tau^+\tau^-$ axes.
- The decay probability

$$\Gamma(H/A \to \tau^+\tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} \pm s_{\perp}^{\tau^+} s_{\perp}^{\tau^-}$$

  is sensitive to the $\tau^{\pm}$ polarization vectors s^{τ^-} and s^{τ^+} (defined in their respective rest frames). The symbols $\parallel,\perp$ denote components parallel/transverse to the Higgs boson momentum as seen from the respective $\tau^{\pm}$ rest frames.
- This idea an its practical refinements are universal: 'Higgs spin' is blind on Higgs origin. But it is not true for the background DY processes .

Important: no dependence on production process

Phenomenology Of Mixed Parity Case

- Higgs boson Yukawa coupling expresed with the help of the scalar–pseudoscalar mixing angle ϕ

$$\bar{\tau} N(\cos\phi + i \sin\phi\gamma_5)\tau$$

- *Decay probability for the mixed scalar–pseudoscalar case*

$$\Gamma(h_{mix} \to \tau^+\tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} + s_{\perp}^{\tau^+} R(2\phi)\, s_{\perp}^{\tau^-}$$

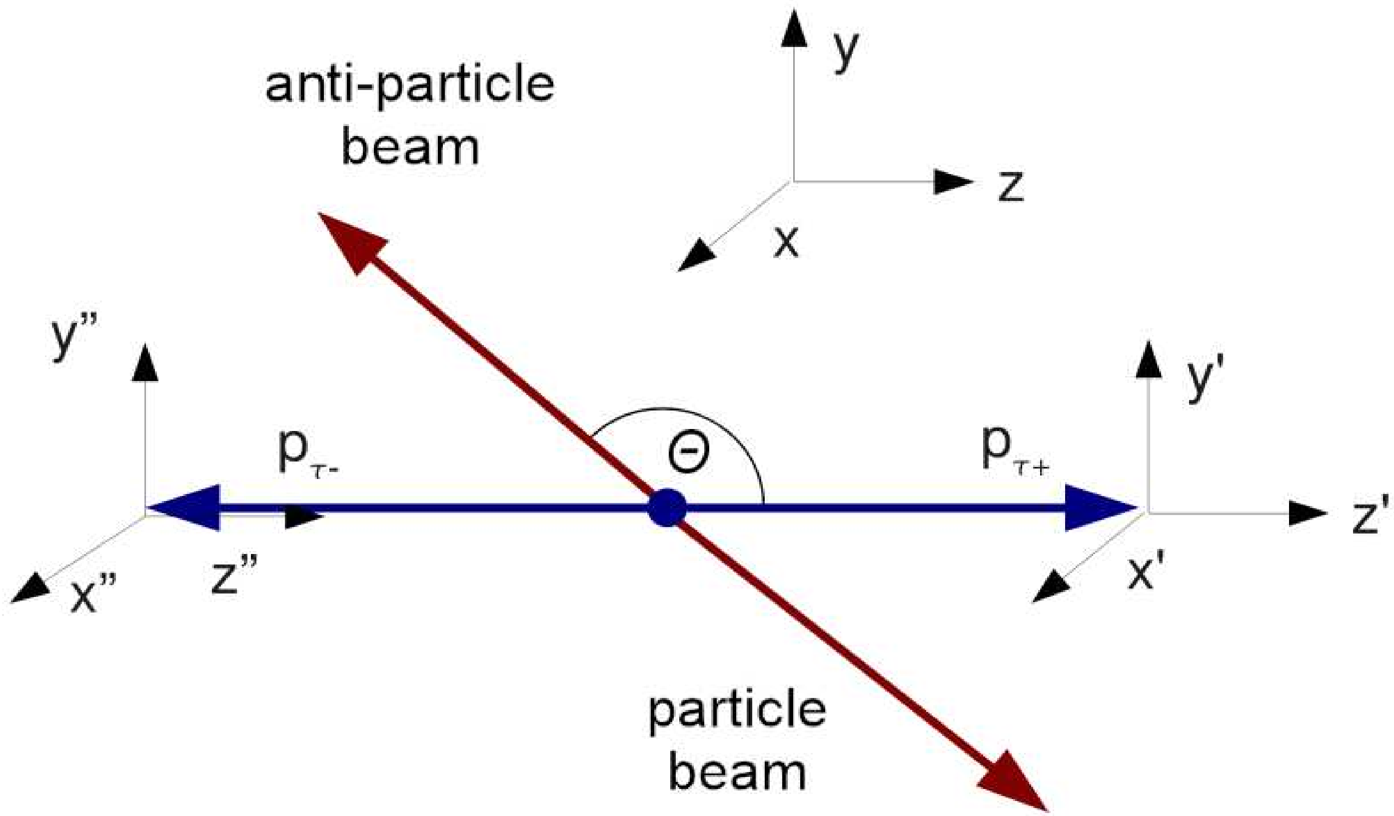
- $R(2\phi) -$ *operator for the rotation by angle* 2ϕ *around the* $\parallel$ *direction.*

$$R_{11} = R_{22} = \cos 2\phi \qquad R_{12} = -R_{21} = \sin 2\phi$$

- *Pure scalar case is reproduced for* $\phi = 0$.
- *For* $\phi = \pi/2$ *we reproduce the pure pseudoscalar case.*

Tau lepton in detector: theorist perspective

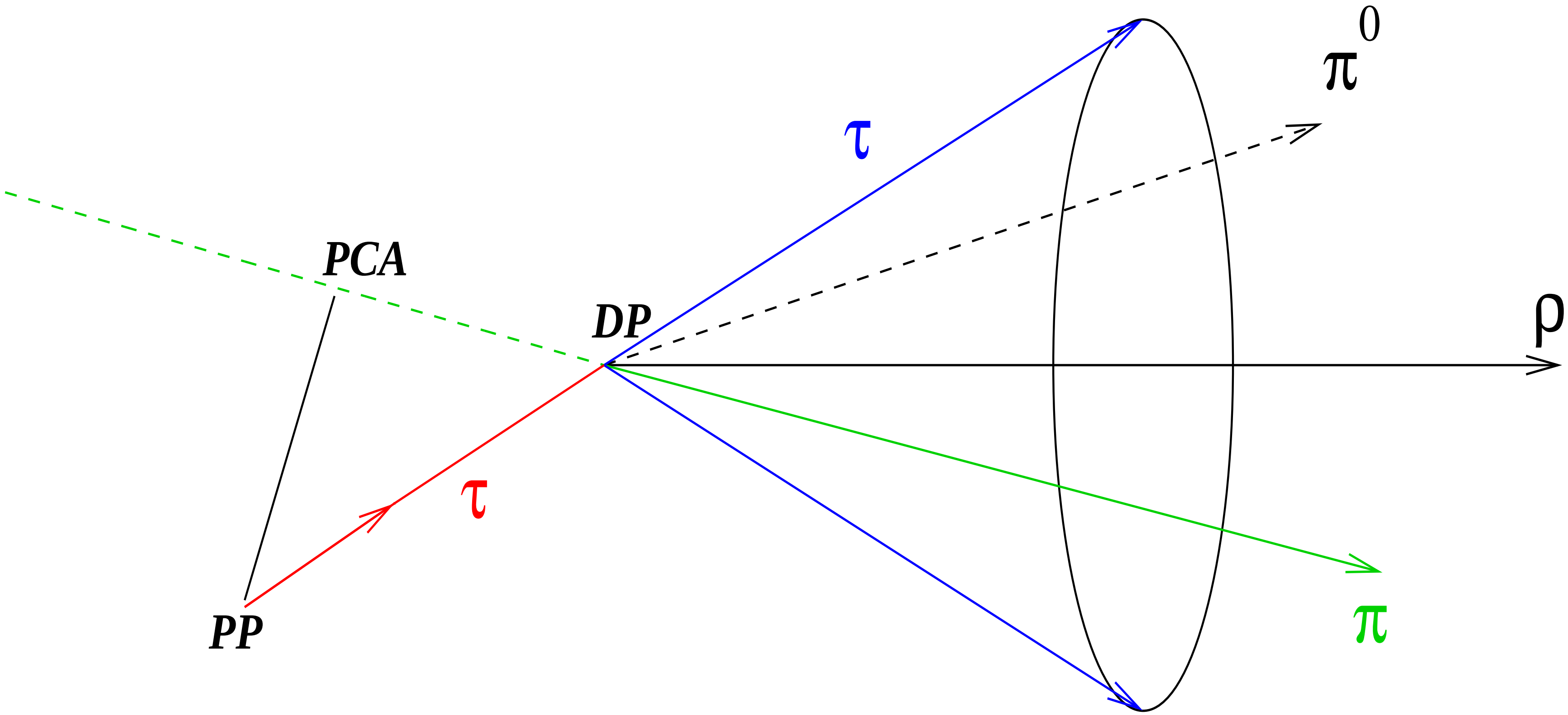
What does it mean τ pair?



Coordinate systems of H (or Z/γ^*), τ^+ and τ^- rest frames

Tau lepton in detector: theorist perspective

What does it mean τ lepton in the detector?



Measurable $\pi^{\pm}, \ \pi^0$,

ν_τ through impact parameter and reconstruction (if at all).

Tau lepton in detector: theorist perspective

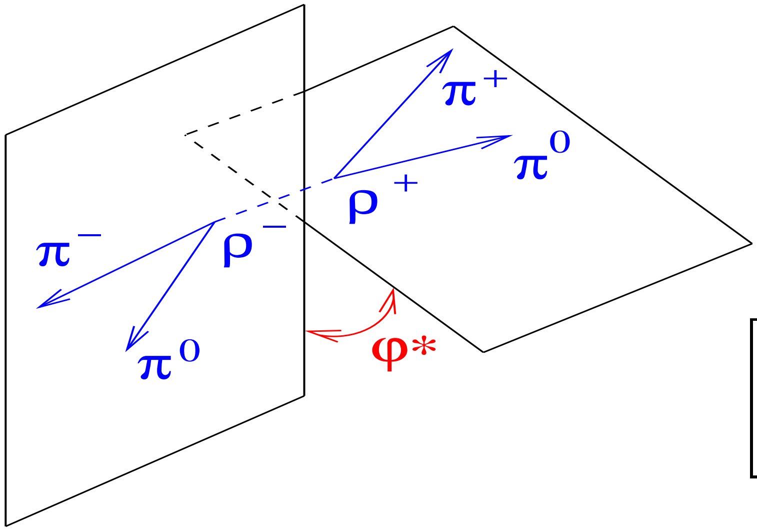
Transverse spin correlations through τ decays

- *Case of $\tau \to \rho\nu_\tau$ decay,* $\mathcal{BR}(\tau \to \rho\nu_\tau) = 25\%$
- *Polarimeter vector h^i is (where q for $\pi^\pm - \pi^0$ and N for ν_τ four momenta.*

$$h^i = \mathcal{N}\Big(2(q\cdot N)q^i - q^2 N^i\Big)$$

$$q\cdot N = (E_{\pi^\pm} - E_{\pi^0})m_\tau$$

- *Acoplanarity of ρ^+ and ρ^- decay prod. (in $\rho^+\rho^-$ r.f.) and events separation.*



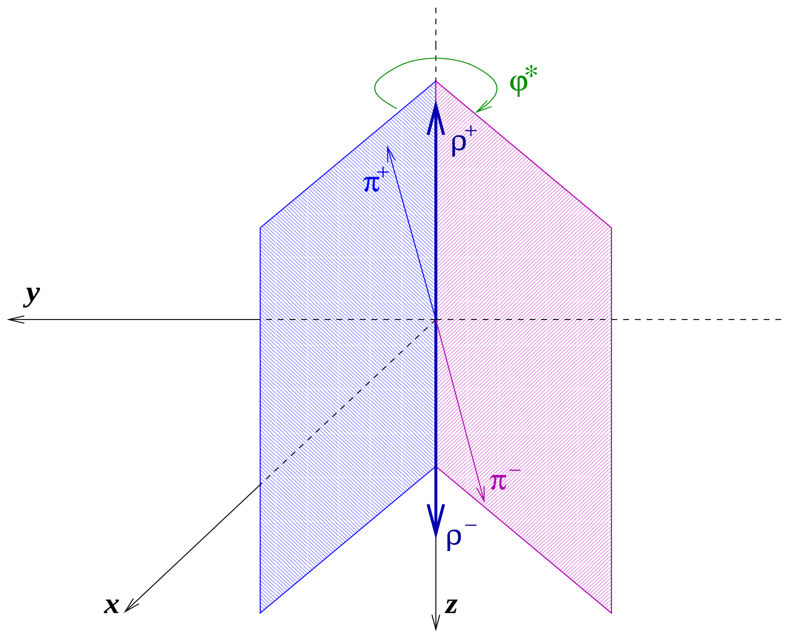
$y_1 y_2 > 0\,;\qquad y_1 y_2 < 0$ *(in $\tau^\pm$ r.f.'s)*

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}\;;\qquad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}.$$

Tau lepton in detector: theorist perspective

Observable For Mixed Scalar–Pseudoscalar Case

- *For mixing angle ϕ, transverse component of τ^+ spin polarization vector is correlated with the one of τ^- rotated by angle 2ϕ.*
- *Acoplanarity $0 < \varphi^* < 2\pi$ is of physical interest, not just $\arccos \mathbf{n}_- \cdot \mathbf{n}_+$.*
- *Distinguish between the two cases $0 < \varphi^* < \pi$ and $2\pi - \varphi^*$*
- *If no separation made the parity effect would wash itself out.*



Normal to planes: $\mathbf{n}_\pm = \mathbf{p}_{\pi^\pm} \times \mathbf{p}_{\pi^0}$

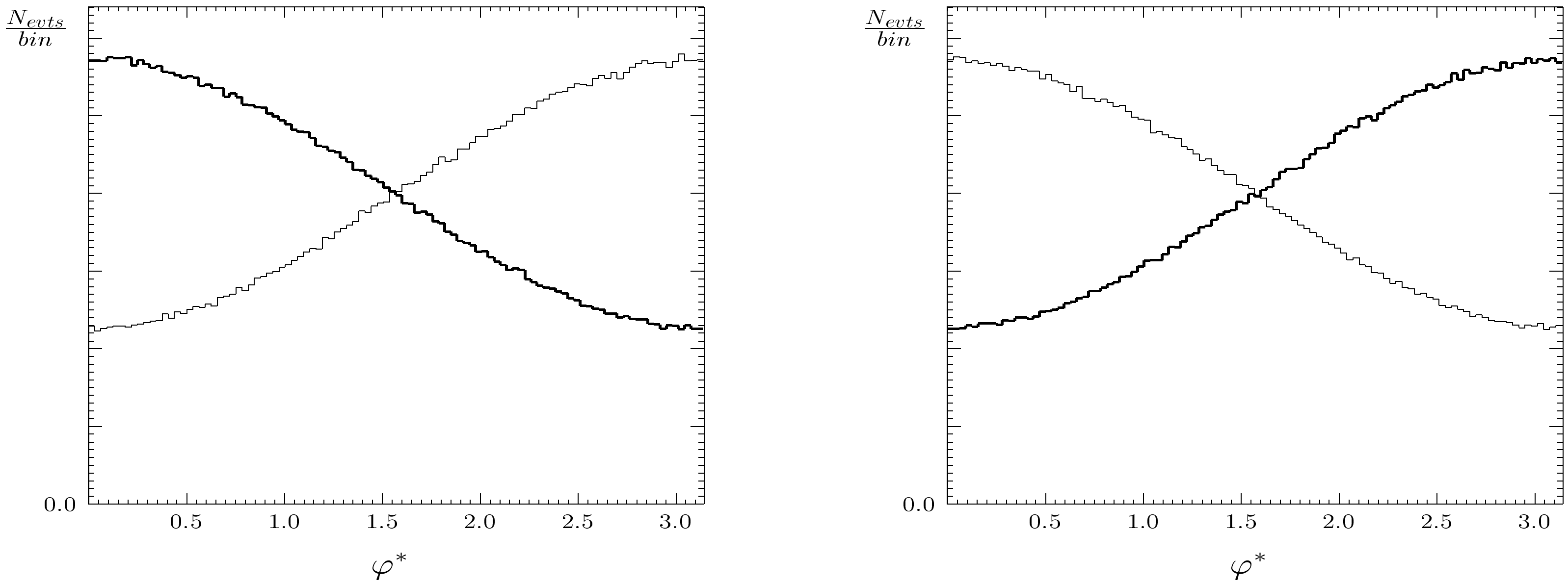
Find the sign of $\mathbf{p}_{\pi^-} \cdot \mathbf{n}_+$

Negative $0 < \varphi^* < \pi$

Otherwise $2\pi - \varphi^*$

Scalar or Pseudoscalar ?

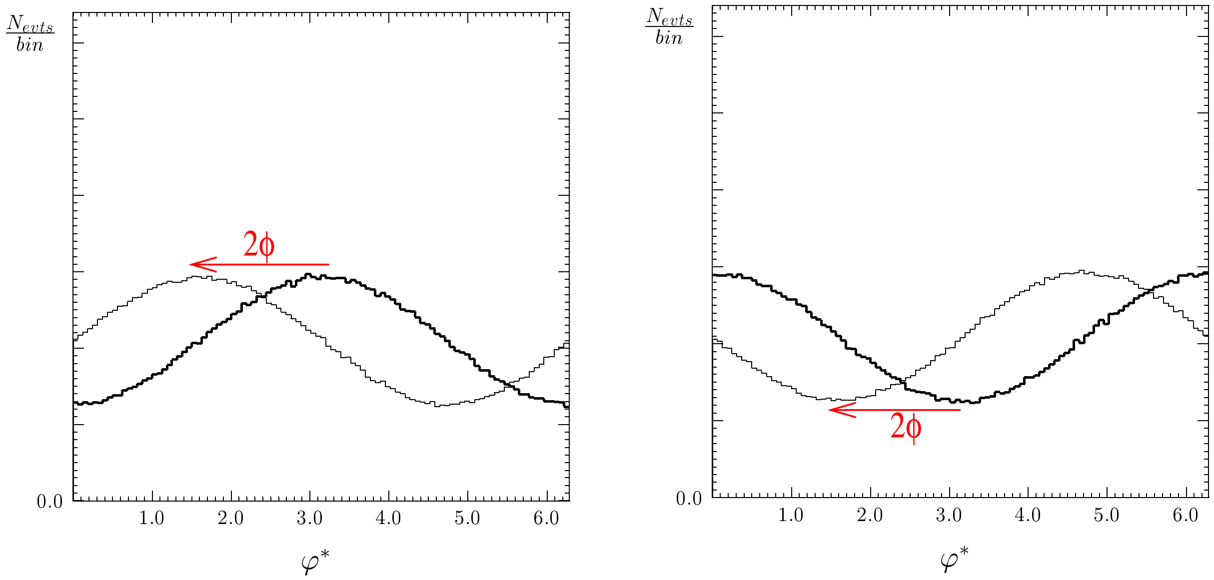
Results Without Smearing



- *The $\rho^+\rho^-$ decay products' acoplanarity distribution without any smearing .*
- *Selection $y_1 y_2 > 0$ is used in the left plot, $y_1 y_2 < 0$ is used for the right plot.*
- *Thick line denote the case of the scalar Higgs and thin lines the pseudoscalar.*
- *Complete spin correlations of $h \to \tau^+\tau^-$, $\tau^\pm \to \rho^\pm \nu$, $\rho^\pm \to \pi^\pm \pi^0$ incl.*

Scalar or Pseudoscalar ?

Old attemps on detector and Impact Parameter



- *Only events where the signs of y1 and y2 are the same whether calculated using the method without or with the help of the τ impact parameter.*
- *Tesla-like set-up SIMDET used,* *K. Desch, A. Imhof, ZW,, M. Worek, Phys.Lett. B579 (2004) 157.*
- *The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one.*

Precision on ϕ $\sim$ 6°, for $1ab^{-1}$ and 350 GeV CMS.

ML results: simulation level: OK

Q: Why ML and not e.g. TMVA?

A: Manpower: in the not so distant past, and now.

I do not plan to cover this topic.

Anyway used by us solutions are outdated they are of nearly wo years ago...

ML results: simulation level: OK

Q: what is ML?

A: I will not cover this topic as well.

I will go to the results

Note:

- From the π^-, π^0 we can define a plane for acoplanarity
- From the π^-, π^-, π^+ we can define four such planes.
- Each plane bring its own y_i variable to avoid cancellations due to properties of τ decay ME: $<\cos\theta>=0$.

ML results: simulation level: OK

Features/variables	$\rho^\pm - \rho^\mp$ $\rho^\pm \to \pi^0 \pi^\pm$	$a_1^\pm - \rho^\mp$ $a_1^\pm \to \rho^0 \pi^\mp,\ \rho^0 \to \pi^+ \pi^-$ $\rho^\mp \to \pi^0 \pi^\mp$	$a_1^\pm - a_1^\mp$ $a_1^\pm \to \rho^0 \pi^\pm,$ $\rho^0 \to \pi^+ \pi^-$
True classification	0.782	0.782	0.782
$\varphi^*_{i,k}$	0.500	0.500	0.500
$\varphi^*_{i,k}$ and y_i, y_k	0.624	0.569	0.536
4-vectors	0.638	0.590	0.557
$\varphi^*_{i,k}$, 4-vectors	0.638	0.594	0.573
$\varphi^*_{i,k}, y_i, y_k$ and m_i^2, m_k^2	0.626	0.578	0.548
$\varphi^*_{i,k}, y_i, y_k, m_i^2, m_k^2$ and 4-vectors	0.639	0.596	0.573

Table 1: Average probability p_i that a model predicts correctly event x_i to be of a type A (scalar), with training being performed for separation between type A and B (pseudo-scalar). Looks beautiful ML do all for us. Really? Let's check details.

Results with detector smearing, also OK

Features				Ideal ± (stat)	Smeared ± (stat) ± (syst)
ϕ^*	4-vec	y_i	m_i		
				$a_1 - \rho$ Decays	
✓	✓	✓	✓	0.6035 ± 0.0005	$0.5923 \pm 0.0005 \pm 0.0002$
✓	✓	✓	-	0.5965 ± 0.0005	$0.5889 \pm 0.0005 \pm 0.0002$
✓	✓	-	✓	0.6037 ± 0.0005	$0.5933 \pm 0.0005 \pm 0.0003$
-	✓	-	-	0.5971 ± 0.0005	$0.5892 \pm 0.0005 \pm 0.0002$
✓	✓	-	-	0.5971 ± 0.0005	$0.5893 \pm 0.0005 \pm 0.0002$
✓	-	✓	✓	0.5927 ± 0.0005	$0.5847 \pm 0.0005 \pm 0.0002$
✓	-	✓	-	0.5819 ± 0.0005	$0.5746 \pm 0.0005 \pm 0.0002$
				$a_1 - a_1$ Decays	
✓	✓	✓	✓	0.5669 ± 0.0004	$0.5657 \pm 0.0004 \pm 0.0001$
✓	✓	✓	-	0.5596 ± 0.0004	$0.5599 \pm 0.0004 \pm 0.0001$
✓	✓	-	✓	0.5677 ± 0.0004	$0.5661 \pm 0.0004 \pm 0.0001$
-	✓	-	-	0.5654 ± 0.0004	$0.5641 \pm 0.0004 \pm 0.0001$
✓	✓	-	-	0.5623 ± 0.0004	$0.5615 \pm 0.0004 \pm 0.0001$
✓	-	✓	✓	0.5469 ± 0.0004	$0.5466 \pm 0.0004 \pm 0.0001$
✓	-	✓	-	0.5369 ± 0.0004	$0.5374 \pm 0.0004 \pm 0.0001$

Table 2: AUC for NN trained to separate scalar and pseudoscalar hypotheses with combinations of input features marked with a ✓. Results in the column labelled "Ideal" are from NNs trained with ideal MC (particle-level simulation). The results in column labelled "Smeared" are from NNs trained with smeared MC.

Features/variables	$\rho^\pm - \rho^\mp$ $\rho^\pm \to \pi^0 \pi^\pm$	$a_1^\pm - \rho^\mp$ $a_1^\pm \to \rho^0\pi^\pm,\ \rho^0 \to \pi^+\pi^-$ $\rho^\mp \to \pi^0 \pi^\mp$	$a_1^\pm - a_1^\mp$ $a_1^\pm \to \rho^0\pi^\pm,$ $\rho^0 \to \pi^+\pi^-$
$\varphi^*_{i,k}$	1	4	16
$\varphi^*_{i,k}$ and y_i, y_k	3	9	24
$\varphi^*_{i,k}$, 4-vectors	25	36	64
$\varphi^*_{i,k}$, y_i, y_k and m_i, m_k	5	13	30
$\varphi^*_{i,k}$, y_i, y_k, m_i, m_k and 4-vectors	29	45	78

Table 3: Dimensionality of the features which may be used in each discussed configuration of the decay modes. Note that in principle $y_i^\pm$, $y_k^\mp$ may be calculated in the rest frame of the resonance pair used to define $\varphi^*_{i,k}$ planes, but in practice, choice of the frames is of no numerically significant effect. We do not distinguish such variants.

New attempt on detector and Impact Parameter

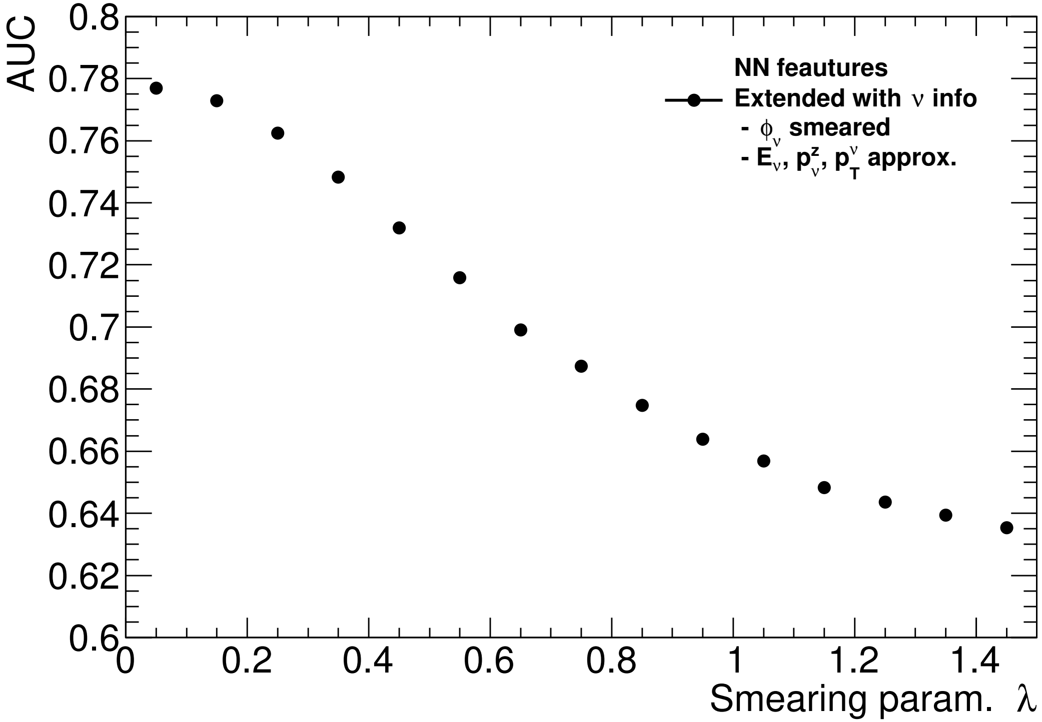


Figure 1: Plot of AUC score for training and validation sample of $\tau\tau$ pairs with one τ decaying $\tau^\pm \to a_1^\pm \nu \to 3\pi^\pm \nu$, second τ decaying $\tau^\pm \to \rho \pm \nu$, as a function of smearing parameter λ, for orientation angle of neutrinos directions around τ hadronic decay products. The rest of kinematic reconstructed from mass or missing p_T constraints.

Dimensionality. No need to worry about complexity. All is perfect.

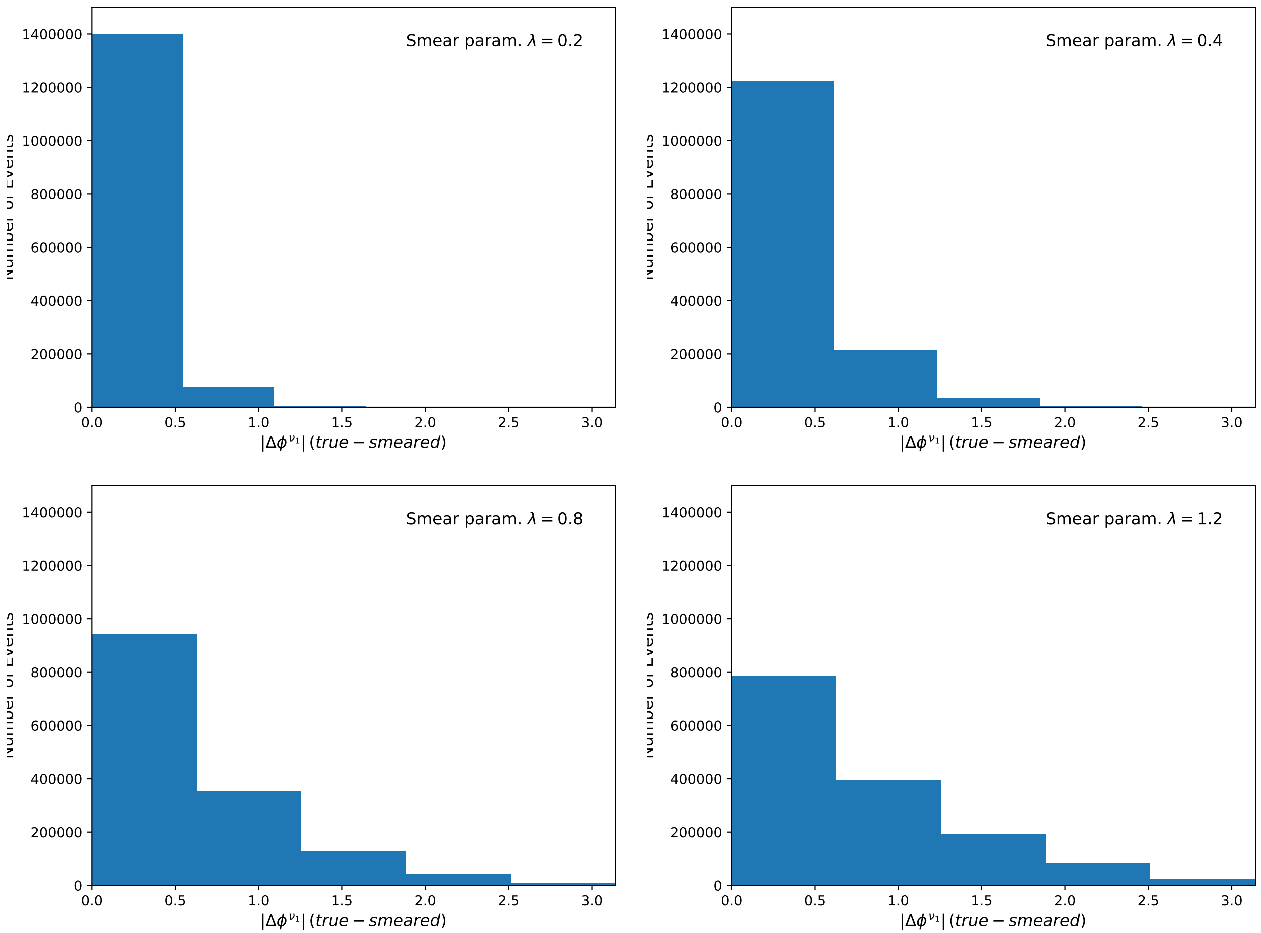


Figure 2: The $|\Delta\phi_{\nu_1}|$ (true - smeared) plot for smearing parameter λ=0.2, 0.4, 0.8 1.2. Note that moderate smearing mean no loss of precision, there is 'critical' range of values, where sensitivity from neutrinos disapear.

Origin of the observables.

- Does it sound like the end of physics?
- I have recalled some results demonstrating sensitivity and its robustness.
- However:
  - (i) Are ML results without hidden conditions?
  - (ii) Are ML results trustworthy?

(i) We had to boost all four momenta into rest-frame of all visible H decay products combined.

(i) We had to align events along the z axis

(i) **Otherwise no sensitivity at all!**

(ii) I will recall definitions and results for classical one dimensional plots.
For the understanding and trust.

- But first: we are not alone with the concerns.

We are not alone ...

- Result depend on model assumptions. Models inspired with results ... Fitting setup $\rightarrow$ biases.
- Our algorithms are far less elaborate than human eye/brain.
- That may look worrisome.

- Biases in art, Giuseppe Arcimboldo (1572 - 1593).

We are not alone ...

Figure 3: Artificial Neural Networks have spurred remarkable recent progress in image classification and speech recognition. But even though these are very useful tools based on well-known mathematical methods, we actually understand surprisingly little of why certain models work and others don't.

From http://googleresearch.blogspot.com/2015/06/inceptionism-going-deeper-into-neural.html

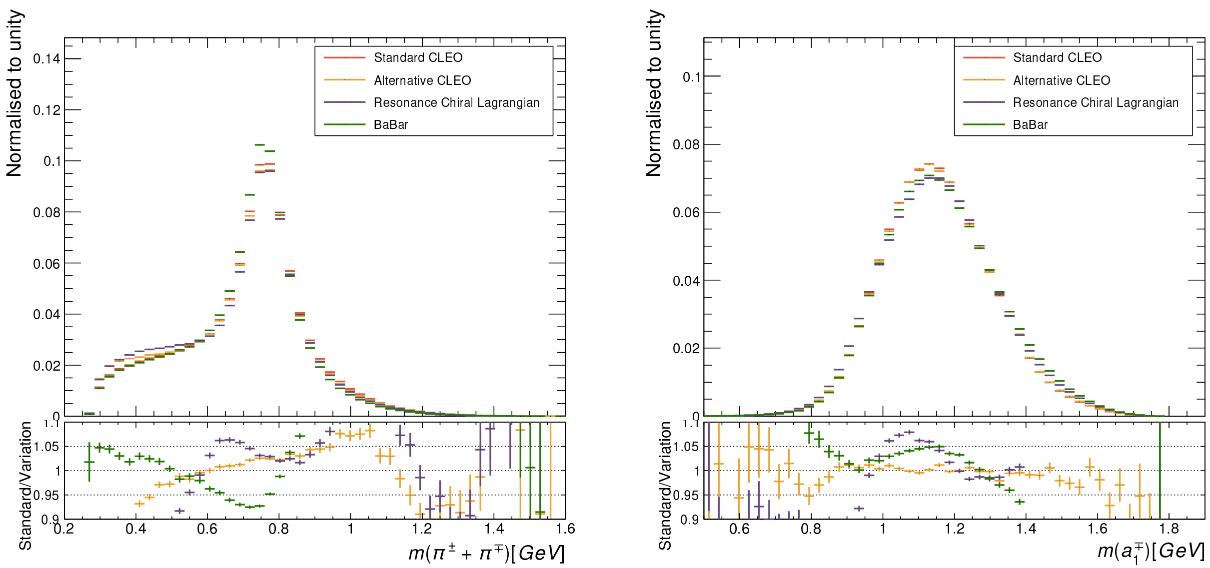
Pattern recognition is an active field and deep concern and not only for us.

Scalar or Pseudoscalar? Human mind

The acoplanarity distributions I have presented for $\rho^+\rho^-$ on slides 10-11, were very close to optimal observables, very useful for interpretation.

They can be useful, even if they are just an element for evaluation of systematic error and to gain confidence that ML results are not too good (to be true).

Four models of hadronic crrents and control dostributions give consistent results, also if different models used for training and analysis for CP classification all went OK.



ML results $\rightarrow$ cross check.

from $\rho^{\pm}$ to $a_1^{\pm}$ case.

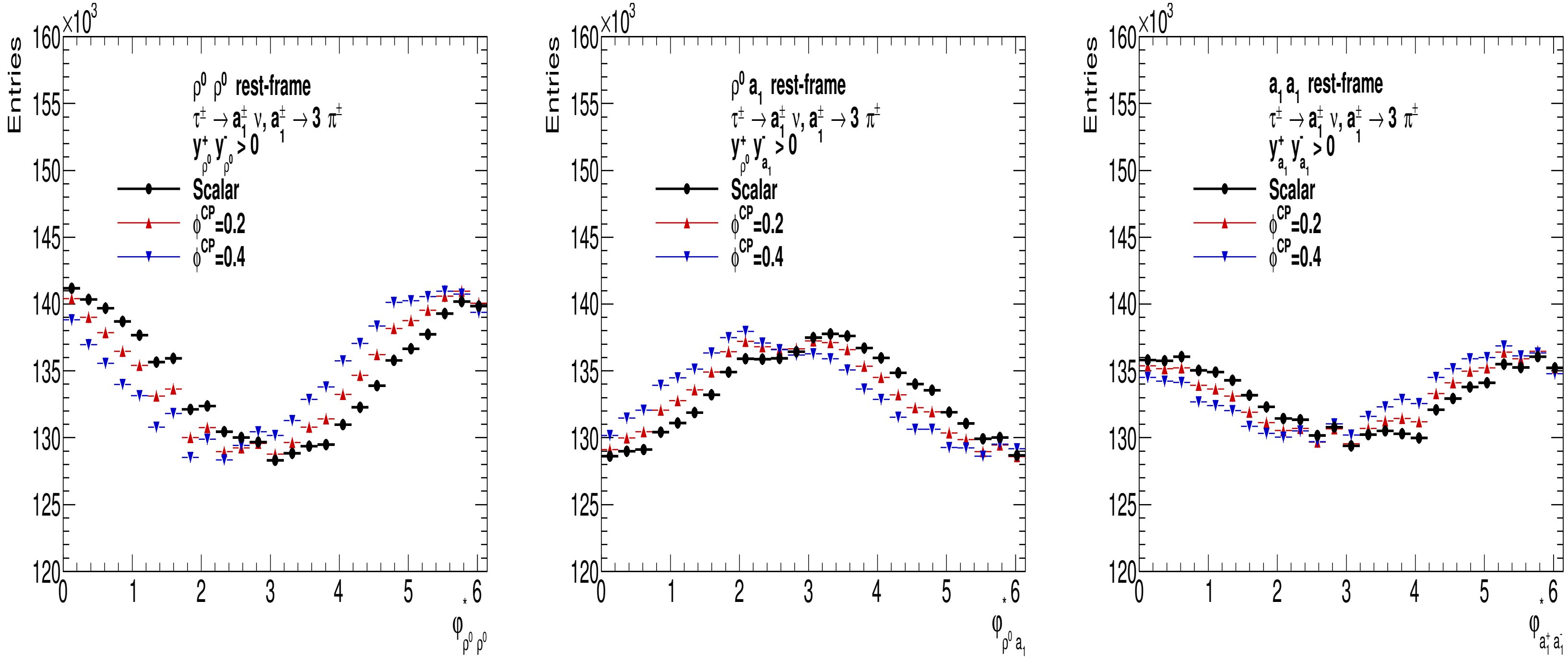
1. In case of $\tau \rightarrow \rho\nu$ there was one decay plane to define and sign of CP sensitive sinusoid was dependent on sign of y_+y_-.
2. In case of $\tau \rightarrow a_1\nu$ four planes can be defined. Two for $a_1 \rightarrow \pi\rho^0$ and another two for $\rho^0 \rightarrow \pi^+\pi^-$ decays.
3. We end up with 4 (or 16) angular distributions and number of y_i like variables.
4. That means meny sub categories to define sample ...
5. All distributions are correlated.
6. Methods of Maschine Learning useful, to evaluate sensitivity of mult-dimensional signatures

   $\rightarrow$ may be more sensitivity can be obtained?

   $\rightarrow$ may be even more sensitivity than possible?

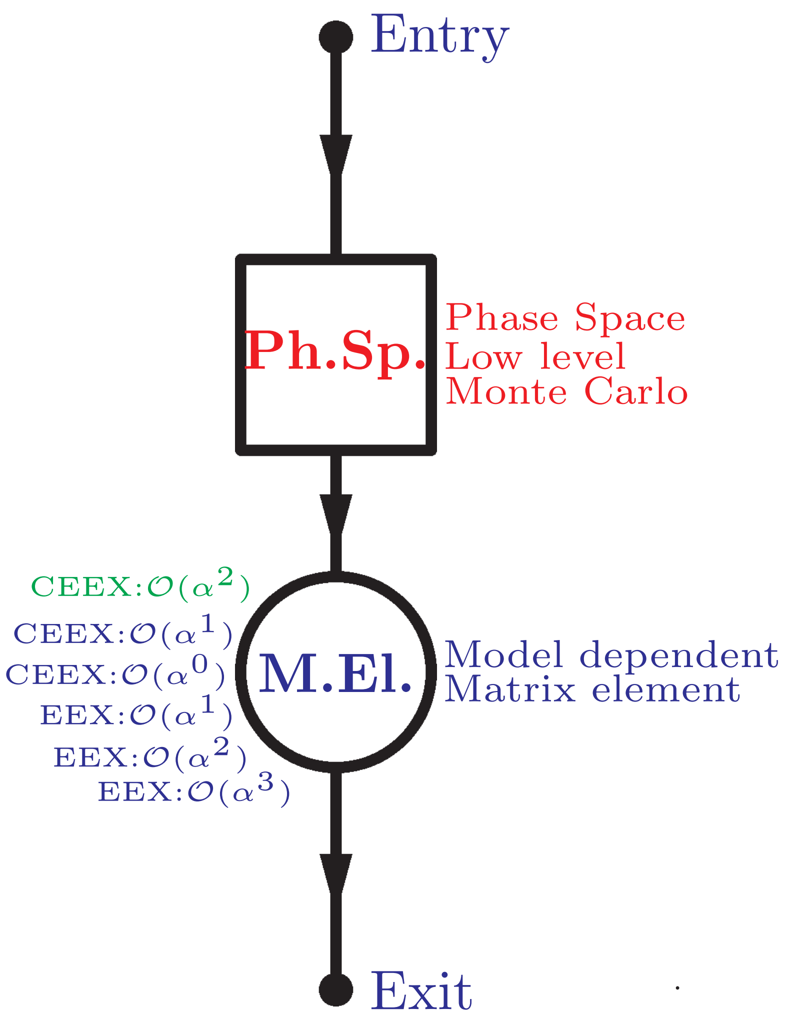
ML results $\longrightarrow$ cross check.

Acoplanarity angles of oriented half decay planes: $\varphi^*_{\rho^0\rho^0}$ (left), $\varphi^*_{a_1\rho^0}$ (middle) and $\varphi^*_{a_1 a_1}$ (right), for events grouped by the sign of $y^+_{\rho^0} y^-_{\rho^0}$, $y^+_{a_1} y^-_{\rho^0}$ and $y^+_{a_1} y^-_{a_1}$ respectively. Three CP mixing angles ϕ^{CP} = 0.0 (scalar), 0.2 and 0.4. Note scale, effect on individual plot is so much smaller now. But up to **16 plots like that** have to be measured, correlations understood. But physics model depends on 1 parameter only and effect of ϕ^{CP}, the Higgs mixing scalar pseudoscalar angle, is always a linear shift.



What we need from Monte Carlo...

Textbook principle "matrix element × full phase space" must be obeyed



- Universal Phase-space Monte Carlo simulator is a separate module producing "raw events" (including importance sampling for possible intermediate resonances)
- Library of several types of models for signal/background provides input for "model weight" which is another independent module
- This is exactly like in case of KORALZ or KKMC of LEP time
- This applies to all information used for ML input too!
- There is plenty of room for biases.
- **Responsablility of physics Monte Carlos → all the following slides**

General formula for tau production and decay.

Formalism for $\tau^+\tau^-$: helps to separate production and decay

- Because narrow τ width approximation can be obviously used for phase space, cross-section for the process $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads:

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

- This formalism is fine, but because of over 20 τ decay channels we have over 400 distinct processes. Also picture of production and decay are mixed.

- Below only τ spin indices are explicitly written:

$$\mathcal{M} = \sum_{\lambda_1 \lambda_2 = 1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\; \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Cross section can be re-written into **core formula of spin algorithms**

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

General formalism for semileptonic decays

- Matrix element used in TAUOLA for semileptonic decay

$$\tau(P,s) \to \nu_\tau(N)X$$
$$\mathcal{M} = \frac{G}{\sqrt{2}}\bar{u}(N)\gamma^\mu(v+a\gamma_5)u(P)J_\mu$$

- J_μ the current depends on the momenta of all hadrons

$$|\mathcal{M}|^2 = G^2\frac{v^2+a^2}{2}(\omega + H_\mu s^\mu)$$
$$\omega = P^\mu(\Pi_\mu - \gamma_{va}\Pi^5_\mu), \qquad h_\mu = H_\mu/\omega$$
$$H_\mu = \frac{1}{M}(M^2\delta^\nu_\mu - P_\mu P^\nu)(\Pi^5_\nu - \gamma_{va}\Pi_\nu)$$
$$\Pi_\mu = 2[(J^*\cdot N)J_\mu + (J\cdot N)J^*_\mu - (J^*\cdot J)N_\mu]$$
$$\Pi^{5\mu} = 2\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho N_\sigma$$
$$\gamma_{va} = -\frac{2va}{v^2+a^2}$$
$$\hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2}m_\nu M(J^*\cdot J)$$
$$\hat{H}^\mu = -2\frac{v^2-a^2}{v^2+a^2}m_\nu\,\mathrm{Im}\,\epsilon^{\mu\nu\rho\sigma}J^*_\nu J_\rho P_\sigma$$

Spin corelation matrix.

Higgs Boson Parity

- *Again scalar/pseudoscalar decay probability in formalism of Kramer et al.*

$$\Gamma(H/A^0 \to \tau^+\tau^-) \sim 1 - s_\parallel^{\tau^+} s_\parallel^{\tau^-} \pm s_\perp^{\tau^+} s_\perp^{\tau^-}$$

- s^τ *is the* τ *polarization vectors.*
- $\parallel$ / $\perp$ *denote components parallel / transverse to the Higgs boson momentum.*
- *The spin weight* $0 < wt < 4$ *is given by the following formula*

$$wt = \frac{1}{4}\left(1 + \sum_{ij=1}^{3} R_{ij} h^i h^j\right)$$

$R_{33} = -1, R(2\phi)$ *defined earlier and for the general case*

Results relevant for fitting and for τ leptons.

- **Applications**

- E. Barberio, B. Le, E. Richter-Was, Z. Was, D. Zanzi and J. Zaremba, “Deep learning approach to the Higgs boson CP measurement in $H \to \tau\tau$ decay and associated systematics,” Phys. Rev. D **96**, 073002 (2017)

- *W production at LHC: lepton angular distributions and reference frames for probing hard QCD*, E. Richter-Was and Z. Was, arXiv:1609.02536

- *Separating electroweak and strong interactions in Drell— Yan processes at LHC: leptons angular distributions and reference frames*, E. Richter-Was and Z. Was, Eur.Phys.J. C76 (2016) 473

- **Tools:**

- *“TauSpinner Program for Studies on Spin Effect in tau Production at the LHC”*, Z. Czyczula, T. Przedzinski and Z. Was, Eur. Phys. J. C **72**, 1988 (2012)

- *“. Production of tau lepton pairs with high pT jets at the LHC and the TauSpinner reweighting algorithm”*, J. Kalinowski, W. Kotlarski, E. Richter-Was and Z. Was, arXiv:1604.00964

Factorizing effective Born from hadronic events

1. We have demonstrated that ML techniques can be useful to distinguish in statistically controllable way between hypotheses of Higgs coupling to tau being CP even, CP-odd or even CP-mix
   for the observables which are massively multi-dimensional.

2. I have pointed issues of mis-interpretations known in the industry.

3. It is known in High energy physics too, e.g. in the domain of jets:
   https://indico.cern.ch/event/667334/
   Advanced Machine Learning for Classification, Regression, and Generation in Jet Physics ,
   Ben Nachman (LBL) CERN Nov 15 2017

4. LESSON: it is important to separate those degrees of freedom which can be controlled, from those where more effort is still needed.

5. In case of signal, that is Higgs production and decay, it is easy: Higgs is narrow and its spin is zero, production is well separated from decay.

6. Problem may come from background. Note that Drell Yan is in comparison huge and intermediate Z state is broader and carries spin.

Let us start with the lowest order coupling constants (without EW corrections) of the Z boson to fermions, $\sin\theta_W^2 = s_W^2 = 1 - m_W^2/m_Z^2$ (on-shell scheme) and T_3^f denotes third component of the isospin.

The vector v_e, v_f and axial a_e, a_f couplings for leptons and quarks are defined with the formulas below:

$$\begin{aligned} v_e &= (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2)/\Delta \\ v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2)/\Delta \\ a_e &= (2 \cdot T_3^e)/\Delta \\ a_f &= (2 \cdot T_3^f)/\Delta \end{aligned} \tag{1}$$

where

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)} \tag{2}$$

properties of Effective Born

With this notation, matrix element for the $q\bar{q} \to Z/\gamma^* \to l^+l^-$, ME_{Born}, can be written as:

$$
\begin{aligned}
ME_{Born} &= [\bar{u}\gamma^\mu v g_{\mu\nu}\bar{v}\gamma^\nu u] \cdot (q_e \cdot q_f) \cdot \frac{\chi_\gamma(s)}{s} \\
&+ [\bar{u}\gamma^\mu v g_{\mu\nu}\bar{\nu}\gamma^\nu u \cdot (v_e \cdot v_f) + \bar{u}\gamma^\mu v g_{\mu\nu}\bar{\nu}\gamma^\nu\gamma^5 u \cdot (v_e \cdot a_f) \\
&+ \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu}\bar{\nu}\gamma^\nu u \cdot (a_e \cdot v_f) + \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu}\bar{\nu}\gamma^\nu\gamma^5 u \cdot (a_e \cdot a_f)] \cdot \frac{\chi_Z(s)}{s}
\end{aligned} \tag{3}
$$

Z-boson and photon propagators read respectively as

$$\chi_\gamma(s) = 1 \tag{4}$$

$$\chi_Z(s) = \frac{G_\mu \dot{M}_z^2}{\sqrt{2} \cdot 8\pi \cdot \alpha_{QED}(0)} \cdot \Delta^2 \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z} \tag{5}$$

At the peak of resonance $|\chi_Z(s)| \times (v_e \cdot v_f) > (q_e \cdot q_f)$ and as a consequence, angular distribution asymmetries of leptons are proportional to $v_e = (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2)$. This gives good sensitivity for s_W^2 measurement.

Above and below resonance we are sensitive to lepton and quark charge instead ...

properties of Effective Born

Born cross-section, for $q\bar{q} \to Z/\gamma^* \to \ell^+\ell^-$ can be expressed as:

$$\frac{d\sigma^{q\bar{q}}_{Born}}{d\cos\theta}(s,\cos\theta,p) = (1+\cos^2\theta)\,F_0(s) + 2\cos\theta\,F_1(s) - p[(1+\cos^2\theta)\,F_2(s) + 2\cos\theta\,F_3(s)] \tag{6}$$

p polarization of the outgoing leptons. The $\cos\theta$ of angle between incoming quark and outgoing lepton in the rest frame of outgoing leptons. All rely on second order spherical harmonics. Also with transverse spin. Form-factors read:

$$\begin{aligned}
F_0(s) &= \frac{\pi\alpha^2}{2s}[q_f^2 q_\ell^2 \cdot \chi_\gamma^2(s) + 2\cdot\chi_\gamma(s) Re\chi_Z(s)\, q_f q_\ell v_f v_\ell + |\chi_Z^2(s)|^2 (v_f^2 + a_f^2)(v_\ell^2 + a_\ell^2)], \\
F_1(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, 2v_f a_f 2v_\ell a_\ell], \qquad (7 \\
F_2(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, (v_f^2 + a_f^2) 2v_\ell a_\ell], \\
F_3(s) &= \frac{\pi\alpha^2}{2s}[2\chi_\gamma(s) Re\chi(s)\, q_f q_\ell v_f v_\ell + |\chi^2(s)|^2\, (v_f^2 + a_f^2) 2v_\ell a_\ell],
\end{aligned}$$

properties of Effective Born

Why is it of interest?

1. Condition: $s_W^2 = 1 - m_W^2/m_Z^2$ is important for some gauge cancellations, in case of multileg processes, but at the same time bring inconsistencies with measurements:
2. either m_W must be off by many experimental errors
3. or electroweak observables such as A_{FB} or P_τ by 50 % of their measurable values.
4. Nonetheless such on mass shell scheme is used by many programs of importance for QCD phenomenology.
5. Technical solutions using calculation of correcting weights are of interest.
6. BY-PRODUCT: separate leptonic degrees of freedom from the hadronic ones.

Effective Born and jets ..

Mustraal frame

[18] F. A. Berends, R. Kleiss, and S. Jadach, *Comput. Phys. Commun.* **29** (1983) 185–200.

Mustraal: Monte Carlo for $e^+ e^- \to \mu^+ \mu^-$ (γ)

$$s = 2p_+ \cdot p_-, \quad t = 2p_+ \cdot q_+, \quad u = 2p_+ \cdot q_-$$

$$s' = 2q_+ \cdot q_-, \quad t' = 2p_- \cdot q_-, \quad u' = 2p_- \cdot q_+$$

$$\sigma_{\rm hard} = \int d\tau (X_i + X_f + X_{\rm int}),$$

The explicit forms of the three terms in $\sigma_{\rm hard}$ read:

$$X_i = \frac{Q^2 \alpha}{4\pi^2 s} \frac{1-\Delta}{k_+ k_-} s'^2 \left[\frac{d\sigma^B}{d\Omega}(s', t, u) + \frac{d\sigma^B}{d\Omega}(s', t', u') \right], \quad (3.4)$$

$$X_f = \frac{Q'^2 \alpha}{4\pi^2 s} \frac{1-\Delta'}{k'_+ k'_-} s^2 \left[\frac{d\sigma^B}{d\Omega}(s, t, u') + \frac{d\sigma^B}{d\Omega}(s, t', u) \right], \quad (3.5)$$

$$X_{\rm int} = \frac{QQ'\alpha}{4\pi^2 s} W \frac{\alpha^2}{2ss'} \left[(u^2 + u'^2 + t^2 + t'^2) \tilde{f}(s, s') + \tfrac{1}{2}(u^2 + u'^2 - t^2 - t'^2) \tilde{g}(s, s') \right]$$
$$+ \frac{QQ'\alpha^3}{4\pi^2 s} \frac{(s-s') M\Gamma}{k_+ k_- k'_+ k'_-} \epsilon_{\mu\nu\rho\sigma} p_+^\mu p_-^\nu q_+^\rho q_-^\sigma \left[\tilde{E}(s, s')(t^2 - t'^2) + \tilde{F}(s, s')(u^2 - u'^2) \right], \quad (3.6)$$

Resulting optimal frame used to minimise higher order corrections from initial state radiation in $e^+e^- \to Z/\gamma^* \to \mu\ \mu$ for algorithms of genuine EW corrections implementation in LEP time Monte Carlo's like Koral Z.

2

Extending definition of Mustraal frame

- We extended this frame to pp -> $l^+ l^-$ j (j) case
  - reconstruct x_1, x_2 of incoming partons from final state kinematics (information on jets used)
  - assume the quark is following x_1 direction (equivalent to what done in CS frame)
  - calculate (θ_1, ϕ_1), (θ_2, ϕ_2) of two Born's, weight with probability calculated not using couplings

$$wt_1 = \frac{E_{p1}^2(1+\cos\theta_1^2)}{E_{p1}^2(1+\cos\theta_1^2)+E_{p2}^2(1+\cos\theta_2^2)}, \qquad wt_2 = \frac{E_{p2}^2(1+\cos\theta_2^2)}{E_{p1}^2(1+\cos\theta_1^2)+E_{p2}^2(1+\cos\theta_2^2)}$$

3

- We can see that distribution is a stochastic sum of Born-like distributions with coefficients which are *positive thus like probabilities*. But it is only QED!

What are the Limitations and Perspectives for case of QCD jets:

- E. Mirkes and J. Ohnemus, “Angular distributions of Drell-Yan lepton pairs at the Tevatron: Order $\alpha - s^2$ corrections and Monte Carlo studies,” PRD **51** (1995) 4891
- R. Kleiss, “Inherent Limitations in the Effective Beam Technique for Algorithmic Solutions to Radiative Corrections,” Nucl. Phys. B **347**, 67 (1990).

If jets are present definition of angles θ, ϕ, of effective Born becomes an **issue**. However, only $\alpha_s^2 \sim 0.01$ corrections to spherical harmonics independently of the choice of reference frame, p_T transverse momentum of $\tau\tau$-pair, Y rapidity:

$$\begin{aligned}\frac{d\sigma}{dp_T^2 dY\, d\cos\theta d\phi} &= \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^2 dY}[(1+\cos^2\theta) + \\ 1/2\, A_0(1-3\cos^2\theta) &+ A_1\sin(2\theta)\cos\phi + 1/2\, A_2\sin^2\theta\cos(2\phi) + A_3\sin\theta\cos\phi \\ +A_4\cos\theta &+ A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\ \sin\phi] \quad (8)\end{aligned}$$

Numerical results, Born recovery?

We will use samples of events generated with the MadGraph matrix element Monte Carlo for Drell-Yan production of τ-lepton pairs, with $m_{\tau\tau} = 80 - 100$ GeV and 13 TeV pp collisions. Lowest order spin amplitudes are used in this program for the parton level process. For the EW scheme we have used default initialisation of MadGraph, with on-shell definition of $\sin^2\theta_W = 1 - m_W^2/m_Z^2 = 0.2222$, which determines value of the axial coupling for leptons and quarks to the Z-boson. The incoming partons are distributed accordingly to PDFs (using CTEQ6L1 PDFs).

Moments method

- We use the Monte Carlo sample of $Z \to \ell^{\pm}\ell^{\mp}$ events and extract angular coefficients of Eq. (8) using moments methods [Mirkes:1994]. The moment of a polynomial $P_i(\cos\theta, \phi)$, integrated over a specific range of p_T, Y defines:

$$\langle P_i(\cos\theta,\phi)\rangle = \frac{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, P_i(\cos\theta,\phi) d\sigma(\cos\theta,\phi)}{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, d\sigma(\cos\theta,\phi)}. \quad (9)$$

- Owing to the orthogonality of the spherical polynomials of Eq. (8), the weighted average of the angular distributions with respect to any specific polynomial, Eq. (9), isolates its corresponding coefficient, averaged over some phase-space region.

- We obtain:

$$\begin{aligned}
\langle \frac{1}{2}(1-3\cos^2\theta)\rangle &= \tfrac{3}{20}(A_0 - \tfrac{2}{3}); & \langle \sin 2\theta \cos\phi\rangle &= \frac{1}{5}A_1;\\
\langle \sin^2\theta\cos 2\phi\rangle &= \tfrac{1}{10}A_2; & \langle \sin\theta\cos\phi\rangle &= \frac{1}{4}A_3;\\
\langle \cos\theta\rangle &= \tfrac{1}{4}A_4; & \langle \sin^2\theta \sin 2\phi\rangle &= \frac{1}{5}A_5;\\
\langle \sin 2\theta\sin\phi\rangle &= \tfrac{1}{5}A_6; & \langle \sin\theta\sin\phi\rangle &= \frac{1}{4}A_7.
\end{aligned} \quad (10)$$

Moments method

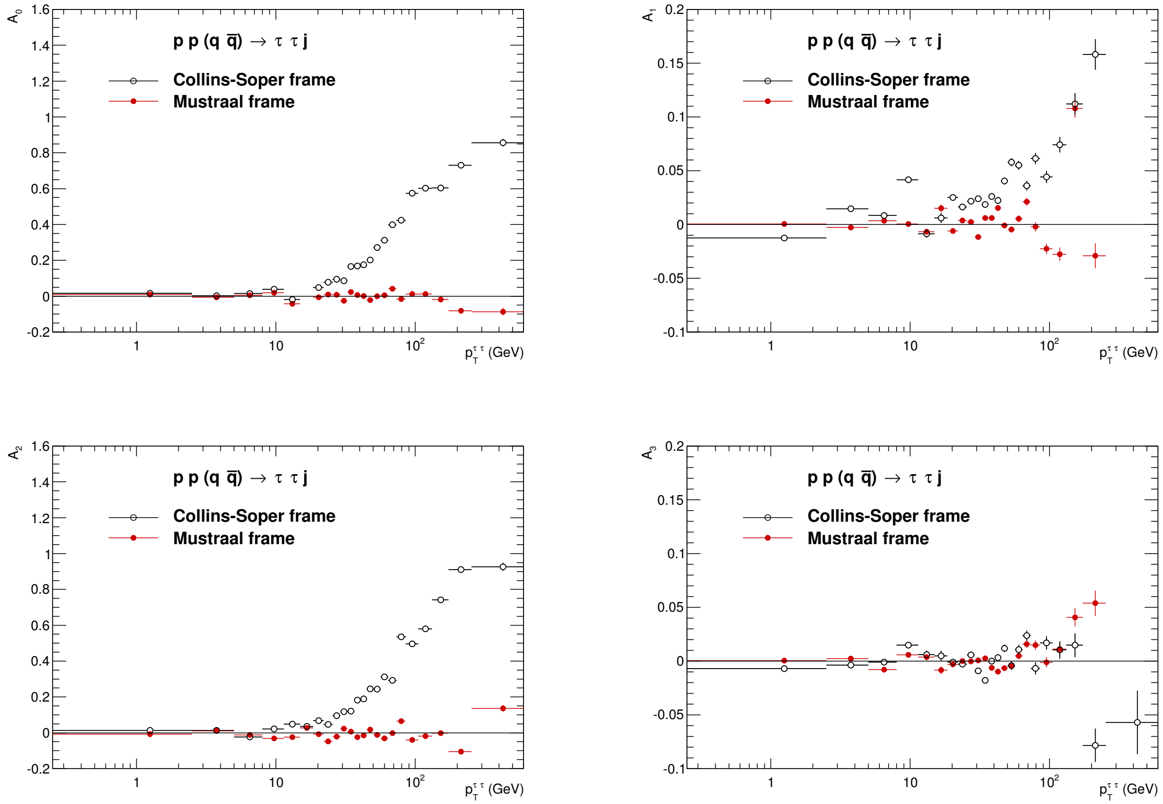


Figure 4: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in Mustraal (red) frames for $pp(q\bar{q}) \to \tau\tau j$ process generated with MadGraph. From **Eur.Phys.J. C76 (2016) 473** . Tree level ME+ collinear pdf's used for analyzed sample.

Mustraal frame works PERFECT. Note that our probablities/weights were stripped from dependence on EW parameters. It could be not so, but IS SO

Moments method

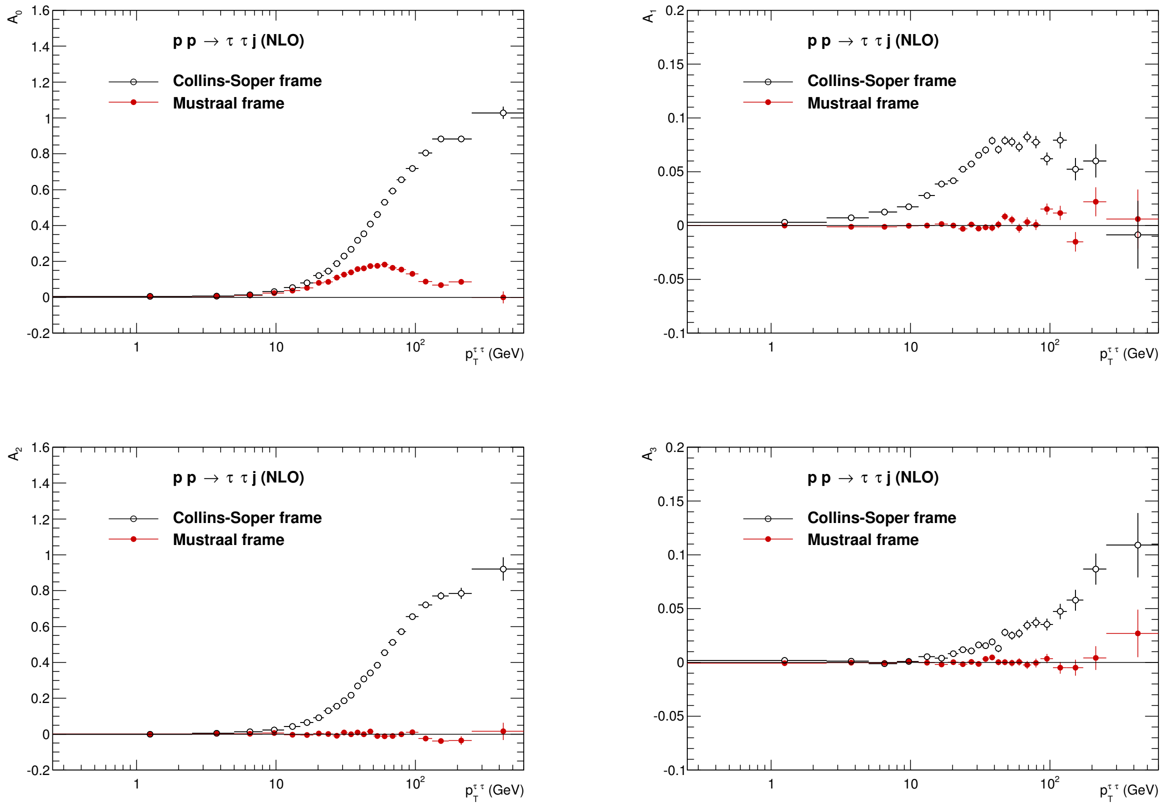


Figure 5: The A_i coefficients of Eq. (8)) calculated in Collins-Soper (black) and in Mustraal (red) frames for $pp \to \tau\tau j$ (NLO) process generated with Powheg+MiNLO. From **Eur.Phys.J. C76 (2016) 473**.

Note that for complete QCD Z+1jet NLO plus MiNLO pattern remained!

Moments method

- The choice of Mustraal frame is result of careful study of single photon (gluon) emission)
- In Ref of 1982 it was shown, that differential distribution is a sum of two born-like distributions convoluted with emission factors.
- This is a consequence of Lorentz group representation and that is why it generalizes to the case of double gluon or even double parton emissions.
- Impact of jets on effective Born is like change of orientation of frames.
- This observation is helpful to separate leptonic degrees of freedom from the ones of hadronic jets, where modelling could bring problems.

Reconstruction for precision observable at LEP

- Still another example, where multi-dimensionality was important for precision
- LEP times precision breakthrough: from 2-3 % on luminosity measurement down to 0.041 %.
- In principle it was just counting experiment.
- Once precision improved, nothing remained simple. Simulation became essential.
- Thanks to introduction into simulation of final states consisting of electron and soft collinear photons...
- **... one could identify that corresponding events was not a detector malfunction.**
- I can not find the appropriate (plastic) slide of that times.

Summary

- I have presented general principle of CP Higgs parity measurement in $H \to \tau\tau$ decays
- I have demonstrated how computer algebraic methods or pattern recognition techniques (Maschine Learning) was useful to manage observable of multi-dimensional nature.
- I have adressed the facorization properties which were helpful to design observables and later to control if NN response was not 'too good'.
- Starting from Higgs decay I have moved to discuss similar factorization properties od DY. They may be also offering a necessary benchmark path for the forthcoming applications.
- I recalled result of LEP sub permille precision level observable. There, details of multidimensional nature were of a great importance.
- **This can be only more true for the future applications. ML learning can enforce importance of multidimensional predictions and necessity of phase-space$\times$ME approach.**

Summary

- Case of Higgs parity is ‘easy’. Dynamic of tau decays is known, Higgs production separates from decay
- case of its background is manageable: Dynamic of tau decays is still known, and Zγ^* production convolutes with decays through spherical harmonics of the second order.
- Case of jet physics is by far more complex, see e.g. afternoon session talks of SMP-J annual workshop, afternoon talks: https://indico.cern.ch/event/684248/
- Managing detctor granurality to improve precision? I think it is good set of topics.