Multi quark production - quantum interference effects

Alex Kovner

University of Connecticut

February 28, 2018

[with Amir Rezaeian Phys.Rev. D96 (2017) no.7, 074018 and arXiv:1801.04875]

Motivation: Correlated production

Observation of Ridge $\rightarrow$ study of correlated particle production.
Most likely several mechanisms are at play.
Final state effects among them, but initial state effects may also be important.
Maybe those can be observed in other systems (not heavy ions/dense protons)?
In this talk effects of quantum statistics on "parton level" :
1. Quantum interference as the origin of "Glasma graph" correlated production. K. Dusling and R. Venugopalan, Phys.Rev. D87 (2013) no.9, 094034 and more; T. Altinoluk, N. Armesto, G. Beuf, A.K., M. Lublinsky, Phys.Lett. B752 (2016) 113-121 and more
2. Exclusive gluon production - model calculation. B. Blok, C. Jakel, M. Strikman, U. Widemann, JHEP 1712 (2017) 074. Also earlier: L. McLerran and V. Skokov, Nucl.Phys. A947 (2016) 142-154

Gluons are hard, quarks are easy

Quantum interference effeects in the “simplest” setting.

“Hybrid approach”: “partonic” proton scatters eikonally on color fields of the target “nucleus”.

Relevant for forward scattering (projectile x values not too small). But qualitative features should be fairly universal

Double quark production

"Forward" production: the quarks come directly from the Proton wave function.

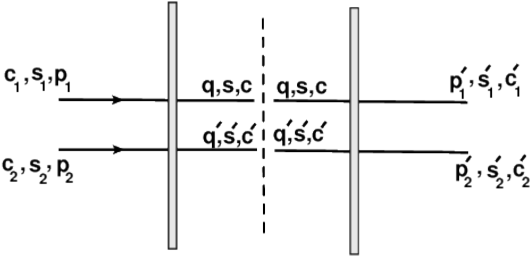

Figure: Two quark production in the background of the CGC field.

$$d\,\sigma^{p+A\to qq+X} = \frac{d^3q}{(2\pi)^3\,2q^-}\frac{d^3q'}{(2\pi)^3\,2q'^-}\,\langle|\langle \text{jet}(q),\text{jet}(q')|\text{Proton}\rangle|^2\rangle_{\text{color sources}}$$

$$|\text{Proton}\rangle = \sum_X\sum_{c_i,s_i}\int_{p_i}\tilde{A}(p_1,c_1,s_1;p_1,c_2,s_2;X)|p_1,c_1,s_1;p_2,c_2,s_2;X\rangle$$

2GTMDs

The cross section:

$$\mathcal{I} = 16\frac{(2\pi)^4}{4}q^- q'^-$$
$$\frac{N_c^2}{N_c^2-1}\int_{\vec{p}_1,\vec{p}_2,\vec{\Delta}}\Big\{\langle D(\vec{p}_1-\vec{q}+\vec{\Delta}/2,2\vec{\Delta})D(\vec{p}_2-\vec{q}'-\vec{\Delta}/2,-2\vec{\Delta})\rangle\Big[T^D_{qq'}(p_1,p_2,\Delta)-\frac{1}{N_c}T^E_{qq'}(p_1,p_2,\Delta)\Big]$$
$$+\ \frac{1}{N_c}\langle Q(\vec{p}_1-\vec{q}+\vec{\Delta}/2,\ \vec{p}_2-\vec{q}'-\vec{\Delta}/2,\vec{\Delta})\rangle\Big[T^E_{qq'}(p_1,p_2,\Delta)-\frac{1}{N_c}T^D_{qq'}(p_1,p_2,\Delta)\Big]\Big\}$$

with "direct" and "color exchange" 2GTMD's

$$T^D_{qq'}(p_1,p_2,\Delta)\equiv\sum_{s_i,c_i}\langle\text{Proton}|\psi^\dagger_q(\vec{p}_1,x_1,c_1,s_1)\psi^\dagger_{q'}(\vec{p}_2,x_2,c_2,s_2)\psi_{q'}(\vec{p}_2-\vec{\Delta},x_2,c_2,s_2)\psi_q(\vec{p}_1+\vec{\Delta},x_1,c_1,s_1)|\text{Proton}\rangle$$
$$T^E_{qq'}(p_1,p_2,\Delta)\equiv\sum_{s_i,c_i}\langle\text{Proton}|\psi^\dagger_q(\vec{p}_1,x_1,c_1,s_1)\psi^\dagger_{q'}(\vec{p}_2,x_2,c_2,s_2)\psi_{q'}(\vec{p}_2-\vec{\Delta},x_2,c_1,s_2)\psi_q(\vec{p}_1+\vec{\Delta},x_1,c_2,s_1)|\text{Proton}\rangle$$

The "dipole" and the "quadrupole" amplitudes:

$$D(\vec{x}_1,\vec{x}_2) \equiv \frac{1}{N_c}Tr\Big[U^\dagger(\vec{x}_1)U(\vec{x}_2)\Big],$$
$$Q(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) \equiv \frac{1}{N_c}Tr\Big[U^\dagger(\vec{x}_1)U(\vec{x}_2)U^\dagger(\vec{x}_3)U(\vec{x}_4)\Big]$$

2GTMD's abridged - nonidentical quarks

Consider production of a u and a d quark.

$$\sum_{ab;s_1s_2}\langle \text{Proton}|\psi^\dagger_u(\vec p_1,x_1,a,s_1)\psi^\dagger_d(\vec p_2,x_2,b,s_2)\psi_d(\vec p_2-\Delta,x_2,b,s_2)\psi_u(\vec p_1+\Delta,x_1,a,s_1)|\text{Proton}\rangle$$
$$=\sum_i\langle \text{Proton}|\sum_{a,s_1}\psi^\dagger_u(\vec p_1,x_1,a,s_1)\psi_u(\vec p_1+\Delta,x_1,a,s_1)|i\rangle\langle i|\sum_{b,s_2}\psi^\dagger_d(\vec p_2,x_2,b,s_2)\psi_d(\vec p_2-\Delta,x_2,b,s_2)|\text{Proton}\rangle$$

Assume a single nucleon "saturates" intermediate states:

$$\sum_{ab;s_1s_2}\langle \text{Proton}|\psi^\dagger_u(\vec p_1,x_1,a,s_1)\psi^\dagger_d(\vec p_2,x_2,b,s_2)\psi_d(\vec p_2-\Delta,x_2,b,s_2)\psi_u(\vec p_1+\Delta,x_1,a,s_1)|\text{Proton}\rangle$$
$$=4N_c^2\mathcal{T}_u(x_1,0,\vec p_1,\Delta)\mathcal{T}^*_d(x_2,0,\vec p_2-\Delta,\Delta),$$

with GTMD's

$$\mathcal{T}_u(x,\eta,\vec p,\vec k)=\frac{1}{2N_c}\sum_{c,s}\langle \text{Proton}|\psi^\dagger_u(\vec p,x,c,s)\psi_u(\vec p+\vec k,x+\eta,c,s)|P,\vec k,\eta\rangle$$

For "color exchange" contribution:

$$\sum_{ab;s_1s_2}\langle \text{Proton}|\psi^\dagger_u(\vec p_1,x_1,a,s_1)\psi^\dagger_d(\vec p_2,x_2,b,s_2)\psi_d(\vec p_2-\Delta,x_2,a,s_2)\psi_u(\vec p_1+\Delta,x_1,b,s_1)|\text{Proton}\rangle$$
$$=\quad -2N_c^2\mathcal{M}(x_1,\eta=x_2-x_1,\vec p_1,\vec p_2-\vec p_1-\Delta)\mathcal{M}^*(x_1,\eta=x_2-x_1,\vec p_1+\Delta,\vec p_2-\vec p_1-\Delta)$$

with "off diagonal" GTMD

$$\mathcal{M}(x,\eta,\vec p,\vec k)=\frac{1}{2N_c}\sum_{c,s}\langle \text{Proton}|\psi^\dagger_u(\vec p,x,c,s)\psi_d(\vec p+\vec k,x+\eta,c,s)|N,\vec k,\eta\rangle$$

Here $|N,\vec k,\eta\rangle$ - the neutron state.

The cross section

In the single nucleon dominance approximation:

$$
\begin{aligned}
\mathcal{I}_{ud} &= 16(4N_c^2)\frac{(2\pi)^4}{4}\delta(p_1^- - q^-)\delta(p_1'^- - q^-)\delta(p_2^- - q'^-)\delta(p_2'^- - q'^-)q^-q'^- \int_{p_1,p_2,\Delta} \\
&\times \Big\{\Big[\langle D(\vec{p}_1 - \vec{q} + \Delta/2, 2\Delta)D(\vec{p}_2 - \vec{q}' - \Delta/2, -2\Delta)\rangle - \frac{1}{N_c^2}\langle Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta)\rangle\Big] \\
&\times \mathcal{T}_u(x_1, 0, \vec{p}_1, \Delta)\mathcal{T}_d^*(x_2, 0, \vec{p}_2, \Delta) \\
&- \frac{1}{N_c}\Big[\langle Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta)\rangle - \langle D(\vec{p}_1 - \vec{q} + \Delta/2, 2\Delta)D(\vec{p}_2 - \vec{q}' - \Delta/2, -2\Delta)\rangle\Big] \\
&\times \mathcal{M}(x_1, x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1 - \Delta)\mathcal{M}^*(x_1, x_2 - x_1, \vec{p}_1 + \Delta, \vec{p}_2 - \vec{p}_1 - \Delta)\Big\}
\end{aligned}
$$

Identical quarks

Antisymmetry of the wave function:

$$\begin{aligned} T^D_{uu}(p_1,p_2,\Delta) &= 4N_c^2\mathcal{T}_u(x_1,\eta=0,\vec{p}_1,\Delta)\mathcal{T}^*_u(x_2,\eta=0,\vec{p}_2,\Delta) \\ &- 2N_c\mathcal{T}_u(x_1,\eta=x_2-x_1,\vec{p}_1,\vec{p}_2-\vec{p}_1-\Delta)\mathcal{T}^*_u(x_1,\eta=x_2-x_1,\vec{p}_1+\Delta,\vec{p}_2-\vec{p}_1-\Delta), \end{aligned}$$

$$\begin{aligned} T^E_{uu}(p_1,p_2,\Delta) &= 4N_c\mathcal{T}_u(x_1,\eta=0,\vec{p}_1,\Delta)\mathcal{T}^*_u(x_2,\eta=0,\vec{p}_2,\Delta) \\ &- 2N_c^2\mathcal{T}_u(x_1,\eta=x_2-x_1,\vec{p}_1,\vec{p}_2-\vec{p}_1-\Delta)\mathcal{T}^*_u(x_1,\eta=x_2-x_1,\vec{p}_1+\Delta,\vec{p}_2-\vec{p}_1-\Delta) \end{aligned}$$

The cross section:

$$\begin{aligned} &\mathcal{I}_{uu} = 16\frac{(2\pi)^4}{4}\delta(p_1^- - q^-)\delta(p_1'^- - q^-)\delta(p_2^- - q'^-)\delta(p_2'^- - q'^-)q^-q'^- \\ &\times 2N_c^2\int_{p_1,p_2,\Delta}\Big[2\langle D(\vec{p}_1-\vec{q}+\Delta/2,2\Delta)D(\vec{p}_2-\vec{q}'-\Delta/2,-2\Delta)\rangle\mathcal{T}_u(x_1,0,\vec{p}_1,\Delta)\mathcal{T}^*_u(x_2,0,\vec{p}_2,\Delta) \\ &-\frac{1}{N_c}\langle Q(\vec{p}_1-\vec{q}+\Delta/2,\vec{p}_2-\vec{q}'-\Delta/2,\Delta)\rangle\mathcal{T}_u(x_1,x_2-x_1,\vec{p}_1,\vec{p}_2-\vec{p}_1-\Delta)\mathcal{T}^*_u(x_1,x_2-x_1,\vec{p}_1+\Delta,\vec{p}_2-\vec{p}_1-\Delta)\Big] \end{aligned}$$

What is the physics of the different terms?

The first term is independent production.

The quadrupole term - correlated production due to quantum statistics effects. "Initial state": Pauli blocking in the incoming wave function; "Final state": HBT effect n the emission.

Quadrupole at large

Target field ensemble is color neutral on distance scales $r \sim 1/Q_s$.
Thus:

$$\lim_{|x_1 - x_3| \gg 1/Q_s} Q(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) = \frac{1}{N_c} \langle U(\vec{x}_1)_{ab} U^\dagger(\vec{x}_2)_{bc} \rangle \langle U(\vec{x}_3)_{cd} U^\dagger(\vec{x}_4)_{da} \rangle$$
$$= D(\vec{x}_1 - \vec{x}_2) D(\vec{x}_3 - \vec{x}_4)$$

And also

$$\lim_{|x_1 - x_2| \gg 1/Q_s} Q(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) = \frac{1}{N_c} \langle U^\dagger(\vec{x}_4)_{da} U(\vec{x}_1)_{ab} \rangle \langle U^\dagger(\vec{x}_2)_{bc} U(\vec{x}_3)_{cd} \rangle$$
$$= D(\vec{x}_1 - \vec{x}_4) D(\vec{x}_2 - \vec{x}_3)$$

Thus for most of configuration space where Q is not small, at leading N_c

$$Q(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) \approx D(\vec{x}_1 - \vec{x}_2) D(\vec{x}_3 - \vec{x}_4) + D(\vec{x}_1 - \vec{x}_4) D(\vec{x}_2 - \vec{x}_3)$$

First term leads to Pauli blocking, second term to quark HBT.

GTMD simplified

We need a qualitatively reasonable model for GTMD.

$$\mathcal{T}_u(x_1, x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1) = T_u(x_1, \vec{p}_1) f(x_1 - x_2) \mathcal{F}(\vec{p}_1 - \vec{p}_2)$$

The form factor

$$\mathcal{F}(\vec{p}) = \frac{1}{(\vec{p}/\Lambda)^2 + 1}, \qquad \text{or} \quad \mathcal{F}(\vec{p}) = e^{-\frac{(\vec{p})^2}{\Lambda^2}}$$

where Λ^{-1} - the "quark" radius of the proton.
For "translationally invariant proton, $\Lambda \to 0$, and $\mathcal{F}(\vec{p}) \sim \delta^2(\vec{p})$

Pauli blocking

The first leading term (assuming translational invariance of the **target**)

$$Q(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) \approx D(\vec{x}_1 - \vec{x}_2) D(\vec{x}_3 - \vec{x}_4))$$

$$Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta) \propto S\delta^2(\Delta) D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}'),$$

Contribution to the cross section:

$$\begin{aligned}\mathcal{I}_{PB} \propto \quad & -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}') |\mathcal{T}_u(x_1, x_2 - x_1, \vec{p}_1, \vec{p}_2 - \vec{p}_1)|^2 \\ & \approx -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}') |\mathcal{T}_u(\vec{p}_1)|^2 |\delta_\Lambda(\vec{p}_2 - \vec{p}_1)\end{aligned}$$

Pauli blocking: initial configurations where two quarks have the same momentum are suppressed.

As expected, not present for non identical quarks.

Hanbury-Brown, Twiss correlation

The second term:

$$Q(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4) \approx D(\vec{x}_1 - \vec{x}_4) D(\vec{x}_3 - \vec{x}_2))$$

$$Q(\vec{p}_1 - \vec{q} + \Delta/2, \vec{p}_2 - \vec{q}' - \Delta/2, \Delta) \propto S\delta^2(\vec{p}_1 - \vec{p}_2 + \Delta + \vec{q} - \vec{q}') D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}'),$$

The contribution to cross section:

$$\begin{aligned} \mathcal{I}_{HBT} \propto \quad & -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}') \mathcal{T}_u(\vec{p}_1, \vec{q}' - \vec{q}) \mathcal{T}_u^*(\vec{p}_2 + \vec{q} - \vec{q}', \vec{q}' - \vec{q}) \\ & \approx -\frac{2}{N_c} S \int_{\vec{p}_1, \vec{p}_2} D(\vec{p}_1 - \vec{q}) D(\vec{p}_2 - \vec{q}') \mathcal{T}_u(\vec{p}_1) \mathcal{T}_u(\vec{p}_2) \delta_\Lambda(\vec{q}' - \vec{q}) \end{aligned}$$

Typical HBT: peaked for equal momenta of final state particles.

Interestingly: there is a analogous correlated term for nonidentical particles with $\mathcal{T} \to \mathcal{M}$.

Other correlations

There are also other sources of correlations in our forlmulae. But they are suppressed by higher power of $1/N_c$:

$$\langle D(\vec{x}_1,\vec{x}_2)D(\vec{x}_3,\vec{x}_4)\rangle \approx \langle D(\vec{x}_1,\vec{x}_2)\rangle\langle D(\vec{x}_3,\vec{x}_4)\rangle + \frac{1}{N_c^2-1}\Big[\langle D(\vec{x}_1,\vec{x}_3)\rangle\langle D(\vec{x}_2,\vec{x}_4)\rangle + \langle D(\vec{x}_1,\vec{x}_4)\rangle\langle D(\vec{x}_2,\vec{x}_3)\rangle\Big]$$

Or suppressed by a power of area, i.e. $S\vec{q}^2$. This is a "classical" contribution from the region

$$|\vec{x}_1-\vec{x}_2| \sim |\vec{x}_1-\vec{x}_3| \sim |\vec{x}_1-\vec{x}_4| \sim Q_s^{-1}$$

Thus quantum statistics effects are very important!

More quarks

It is important to understand what are similar effects in multiparticle production (more than two).

For example, is there a contribution to $v_2(4)$?

So we generalize:

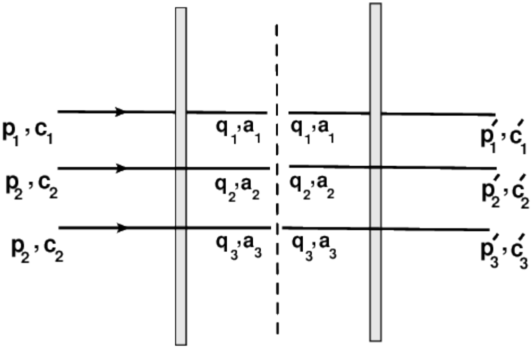

Figure: The diagram contributing to three quark production in the background of the CGC field.

What changes?

$$\mathcal{I} \propto \int_{\mathbf{p}_i,\mathbf{p}'_i,\mathbf{x}_i} e^{i[(\mathbf{p}'_1-\mathbf{q}_1)\mathbf{x}_1+(\mathbf{q}_1-\mathbf{p}_1)\mathbf{x}'_1+(\mathbf{p}'_2-\mathbf{q}_2)\mathbf{x}_2+(\mathbf{q}_2-\mathbf{p}_2)\mathbf{x}'_2]+(\mathbf{p}'_3-\mathbf{q}_3)\mathbf{x}_3+(\mathbf{q}_3-\mathbf{p}_3)\mathbf{x}'_3]}$$
$$\times\langle[U^\dagger(\mathbf{x}_1)]_{c'_1a_1}[U(\mathbf{x}'_1)]_{a_1c_1}[U^\dagger(\mathbf{x}_2)]_{c'_2a_2}[U(\mathbf{x}'_2)]_{a_2c_2}[U^\dagger(\mathbf{x}_3)]_{c'_3a_3}[U(\mathbf{x}'_3)]_{a_3c_3}\rangle$$
$$\sum_X A(p_1,c_1;p_2,c_2;p_3,c_3;X)A^*(p'_1,c'_1;p'_2,c'_2;p'_3,c'_3,X). \tag{1}$$

3GTMD's and more multipole averages:

$$D(\mathbf{x}_1,\mathbf{x}'_1) \equiv \frac{1}{N_c}Tr\left[U^\dagger(\mathbf{x}_1)U(\mathbf{x}'_1)\right],$$
$$Q(\mathbf{x}_1,\mathbf{x}'_1,\mathbf{x}_2,\mathbf{x}'_2) \equiv \frac{1}{N_c}Tr\left[U^\dagger(\mathbf{x}_1)U(\mathbf{x}'_1)U^\dagger(\mathbf{x}_2)U(\mathbf{x}'_2)\right],$$
$$X(\mathbf{x}_1,\mathbf{x}'_1,\mathbf{x}_2,\mathbf{x}'_2,\mathbf{x}_3,\mathbf{x}'_3) \equiv \frac{1}{N_c}Tr\left[U^\dagger(\mathbf{x}_1)U(\mathbf{x}'_1)U^\dagger(\mathbf{x}_2)U(\mathbf{x}'_2)U^\dagger(\mathbf{x}_3)U(\mathbf{x}'_3)\right],$$

Factorize 3GTMD's

In general we need:

$$T^3_{ijk} \equiv \langle P|\psi^{\dagger a}(\mathbf{p}_1)\psi^a(\mathbf{p}'_i)\psi^{\dagger b}(\mathbf{p}_2)\psi^b(\mathbf{p}'_j)\psi^{\dagger c}(\mathbf{p}_3)\psi^c(\mathbf{p}'_k)|P\rangle. \qquad (2)$$

Like for two quarks employ the GTMD factorization approximation (somewhat generalized). For three identical quarks:

$$\begin{aligned} T^3_{ijk} = &\; T(\mathbf{p}_1,\mathbf{p}'_i)T(\mathbf{p}_2,\mathbf{p}'_j)T(\mathbf{p}_3,\mathbf{p}'_k) \\ &-\frac{1}{N_c}\Big[T(\mathbf{p}_1,\mathbf{p}'_i)T(\mathbf{p}_2,\mathbf{p}'_k)T(\mathbf{p}_3,\mathbf{p}'_j) + T(\mathbf{p}_1,\mathbf{p}'_j)T(\mathbf{p}_2,\mathbf{p}'_i)T(\mathbf{p}_3,\mathbf{p}'_k) \\ &+T(\mathbf{p}_1,\mathbf{p}'_k)T(\mathbf{p}_2,\mathbf{p}'_j)T(\mathbf{p}_3,\mathbf{p}'_i)\Big] \\ &+\frac{1}{N_c^2}\Big[T(\mathbf{p}_1,\mathbf{p}'_j)T(\mathbf{p}_2,\mathbf{p}'_k)T(\mathbf{p}_3,\mathbf{p}'_i) + T(\mathbf{p}_1,\mathbf{p}'_k)T(\mathbf{p}_2,\mathbf{p}'_i)T(\mathbf{p}_3,\mathbf{p}'_j)\Big]. \end{aligned}$$

The multipoles

Keep only terms unsuppressed by factors of area:

$$\langle Q(1,1',2,2')\rangle = \bar{Q}(1,1',2,2') + \langle D(1,1')\rangle\langle D(2,2')\rangle + \langle D(1,2')\rangle\langle D(2,1')\rangle.$$

$$X(1,1',2,2',3,3') = \bar{X}(1,1',2,2',3,3') +$$
$$D(1,1')D(2,2')D(3,3') + D(1,1')D(3,3')D(3,2') + D(1,3')D(2,2')D(3$$
$$+D(1,3')D(2,2')D(3,1') + D(1,2')D(2,1')D(3,3')$$
$$+D(1,1')\bar{Q}(2,2',3,3') + D(1,3')\bar{Q}(2,2',3,1') + D(2,1')\bar{Q}(1,2',3,3')$$
$$+D(2,2')\bar{Q}(1,1',3,3') + D(3,2')\bar{Q}(1,1',2,3') + D(3,3')\bar{Q}(1,1',2,2').$$

Only the dipole terms need to be kept!

Leading contributions:

$$I_0 = \int_{\mathbf{p}_i} D(\mathbf{q}_1 - \mathbf{p}_1) D(\mathbf{q}_2 - \mathbf{p}_2) D(\mathbf{q}_3 - \mathbf{p}_3) T(\mathbf{p}_1, \mathbf{p}_1) T(\mathbf{p}_2, \mathbf{p}_2) T(\mathbf{p}_3, \mathbf{p}_3),$$

$$I_1 = -\frac{1}{2N_c} \int_{\mathbf{p}_i} D(\mathbf{q}_1 - \mathbf{p}_1) D(\mathbf{q}_2 - \mathbf{p}_2) D(\mathbf{q}_3 - \mathbf{p}_3) \times$$

$$\Big[T(\mathbf{p}_1, \mathbf{p}_2) T(\mathbf{p}_2, \mathbf{p}_1) T(\mathbf{p}_3, \mathbf{p}_3) + T(\mathbf{p}_1, \mathbf{p}_1) T(\mathbf{p}_2, \mathbf{p}_3) T(\mathbf{p}_3, \mathbf{p}_2)$$

$$+ T(\mathbf{p}_1, \mathbf{p}_3) T(\mathbf{p}_2, \mathbf{p}_2) T(\mathbf{p}_3, \mathbf{p}_1)$$

$$+ T(\mathbf{p}_1, \mathbf{p}_1) T(\mathbf{p}_2, \mathbf{p}_2 + \mathbf{q}_3 - \mathbf{q}_2) T(\mathbf{p}_3, \mathbf{p}_3 + \mathbf{q}_2 - \mathbf{q}_3)$$

$$+ T(\mathbf{p}_1, \mathbf{p}_1 + \mathbf{q}_3 - \mathbf{q}_1) T(\mathbf{p}_2, \mathbf{p}_2) T(\mathbf{p}_3, \mathbf{p}_3 + \mathbf{q}_1 - \mathbf{q}_3)$$

$$+ T(\mathbf{p}_1, \mathbf{p}_1 + \mathbf{q}_2 - \mathbf{q}_1) T(\mathbf{p}_2, \mathbf{p}_2 + \mathbf{q}_1 - \mathbf{q}_2) T(\mathbf{p}_3, \mathbf{p}_3) \Big],$$

$$I_2 = \frac{1}{4N_c^2}\int_{\mathbf{p}_i} D(\mathbf{q}_1-\mathbf{p}_1)D(\mathbf{q}_2-\mathbf{p}_2)D(\mathbf{q}_3-\mathbf{p}_3)\times$$

$$\Big[T(\mathbf{p}_1,\mathbf{p}_2)T(\mathbf{p}_2,\mathbf{p}_3)T(\mathbf{p}_3,\mathbf{p}_1) + T(\mathbf{p}_1,\mathbf{p}_3)T(\mathbf{p}_2,\mathbf{p}_1)T(\mathbf{p}_3,\mathbf{p}_2)$$

$$+T(\mathbf{p}_1,\mathbf{p}_3+\mathbf{q}_2-\mathbf{q}_3)T(\mathbf{p}_2,\mathbf{p}_2+\mathbf{q}_3-\mathbf{q}_2)T(\mathbf{p}_3,\mathbf{p}_1) + T(\mathbf{p}_1,\mathbf{p}_2+\mathbf{q}_3-\mathbf{q}$$

$$+T(\mathbf{p}_1,\mathbf{p}_1+\mathbf{q}_2-\mathbf{q}_1)T(\mathbf{p}_2,\mathbf{p}_3)T(\mathbf{p}_3,\mathbf{p}_2+\mathbf{q}_1-\mathbf{q}_2) + T(\mathbf{p}_1,\mathbf{p}_3)T(\mathbf{p}_2,\mathbf{p}_2$$

$$+T(\mathbf{p}_1,\mathbf{p}_2)T(\mathbf{p}_2,\mathbf{p}_1+\mathbf{q}_2-\mathbf{q}_1)T(\mathbf{p}_3,\mathbf{p}_3+\mathbf{q}_3-\mathbf{q}_2) + T(\mathbf{p}_1,\mathbf{p}_1+\mathbf{q}_3-\mathbf{q}$$

$$+T(\mathbf{p}_1,\mathbf{p}_1+\mathbf{q}_3-\mathbf{q}_1)T(\mathbf{p}_2,\mathbf{p}_2+\mathbf{q}_1-\mathbf{q}_2)T(\mathbf{p}_3,\mathbf{p}_3+\mathbf{q}_2-\mathbf{q}_3)$$

$$+T(\mathbf{p}_1,\mathbf{p}_1+\mathbf{q}_2-\mathbf{q}_1)T(\mathbf{p}_2,\mathbf{p}_2+\mathbf{q}_3-\mathbf{q}_2)T(\mathbf{p}_3,\mathbf{p}_3+\mathbf{q}_1-\mathbf{q}_3)\Big].$$

How to interpret?

We take the same form of GTMD:

$$T(\mathbf{p},\mathbf{k}) = T(\frac{\mathbf{p}+\mathbf{k}}{2})F(\mathbf{k}-\mathbf{p}); \qquad F(\mathbf{k}) = \frac{1}{\frac{\mathbf{k}^2}{\Lambda^2}+1}.$$

In I_1 - nothing new. Pairwise correlations (either HBT or PB) between two of the produced particles.

HBT large when **final observed momenta** are similar $\mathbf{q}_i \sim \mathbf{q}_j$.

PB large when momenta of **incoming** quarks are similar: $\mathbf{p}_i \sim \mathbf{p}_j$.

"Unitarization" corrections

In I_2: the first type

$$\int_{\mathbf{p}_i} D(\mathbf{q}_1 - \mathbf{p}_1) D(\mathbf{q}_2 - \mathbf{p}_2) D(\mathbf{q}_3 - \mathbf{p}_3) T(\mathbf{p}_1, \mathbf{p}_2) T(\mathbf{p}_2, \mathbf{p}_3) T(\mathbf{p}_3, \mathbf{p}_1).$$

Opposite sign to I_1: an indication that I_1 "oversubtracts" the Pauli blocking contribution when all three quarks in the proton wave function have equal momenta.

The positive contribution from I_2 rectifies this "over subtraction".

Second type:

$$\int_{p_i} D(\mathbf{q}_1 - \mathbf{p}_1) D(\mathbf{q}_2 - \mathbf{p}_2) D(\mathbf{q}_3 - \mathbf{p}_3) T(\mathbf{p}_1, \mathbf{p}_1 + \mathbf{q}_2 - \mathbf{q}_1) T(\mathbf{p}_2, \mathbf{p}_2 + \mathbf{q}_3 - \mathbf{q}_2) T(\mathbf{p}_3, \mathbf{p}_3 + \mathbf{q}_1 - \mathbf{q}_3).$$

"Unitarization correction" to the pairwise HBT interference term.

The last type:

$$\int_{\mathbf{p}_i} D(\mathbf{q}_1 - \mathbf{p}_1) D(\mathbf{q}_2 - \mathbf{p}_2) D(\mathbf{q}_3 - \mathbf{p}_3) T(\mathbf{p}_1, \mathbf{p}_1 + \mathbf{q}_3 - \mathbf{q}_1) T(\mathbf{p}_2, \mathbf{p}_3 + \mathbf{q}_2 - \mathbf{q}_3) T(\mathbf{p}_3, \mathbf{p}_2).$$

"Mixed" HBT-PB correction. The magnitude of this term is maximal when $\mathbf{q}_3 = \mathbf{q}_1 = \mathbf{q}_2$ and $\mathbf{p}_2 = \mathbf{p}_3$.

Even more quarks

Area unsupressed terms can be generalized to N-quark production in a straightforward manner.

Very same structure to $O(1/N_c^2)$ plus correlations from more than one pair.

However the number of correlated terms grows fast with N - so age N_c approximation becomes suspect very quickly.

Conclusions

Quantum statistics (quantum interference) effects are ubiquitous. For quark production they are parametrically leading source for correlations. For gluons they are as important as “classical” effects.

Perhaps they can be observed in simpler processes. Semi inclusive DIS?