IFJ PAN
Theory Division – Particle Theory
Sebastian Sapeta and Rene Poncelet

Contact:

Rene Poncelet, e-mail: `rene.poncelet@ifj.edu.pl`
Sebastian Sapeta, e-mail: `sebastian.sapeta@ifj.edu.pl`

Quantum Field Theory

Exercises 3

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3 Wick's theorem

1. *Normal ordering*

   (a) Write the following product of field operators in terms of normal ordered and contracted products of field operators using Wick's theorem

   $$\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \tag{3.1}$$

2. *Feynman graphs*

   (a) Let's assume scalar field theory with ϕ^4 interactions, i.e. the following Lagrangian density

   $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \,. \tag{3.2}$$

   Draw all "Feynman"-diagrams (in position space representation) contributing to 4-point correlation function:

   $$\langle\Omega|\, \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\, |\Omega\rangle \tag{3.3}$$

   at λ^0, λ^1, λ^2 and λ^3 in perturbation theory.
   Hint: Besides using the intuition of what diagrams can contribute you can start from the Gellmann-Low formula

   $$\langle\Omega|\, \phi(x_1)\dots\phi(x_n)\, |\Omega\rangle = \langle 0|\, T\left\{\phi(x_1)\dots\phi(x_n)\exp\left(-i\int \mathrm{d}^4z : \mathcal{H}_{\mathrm{int}}(z) :\right)\right\}|0\rangle\,\Big|_{\text{connected graphs}} \tag{3.4}$$

   and expand in powers of λ. Rewrite, using Wick's theorem, the obtained time-ordered expression in terms of fully contracted contributions.

   (b) Write down the numerical prefactor for each of the diagrams in a).