MC Simulation of the Constrained Markovian Evolution

Loops and Legs 2004

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and

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The name of the game

$$\boxed{(Loops + Legs)^{\infty}}$$

The long standing problem

- Markovian MC implementing the QCD/QED evolution equations is basic ingredient in all parton shower type MCs

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- Backward Markovian does not solve evolution eqs. It merely exploits their solutions coming from the external non-MC methods
- **Is it possible to invent an efficient MC algorithm for constrained Markovian based on *internal* MC solutions of the evolution eqs?**

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Multicomponent evolution equation

$$\frac{\partial}{\partial t} D_k(t,x) = \sum_j \int_x^1 \frac{dz}{z} P_{kj}(z) \frac{\alpha_S(t,z)}{\pi} D_j\Big(t, \frac{x}{z}\Big)$$

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$$= \sum_j \frac{\alpha_S(t,\cdot)}{\pi} P_{kj}(\cdot) \otimes D_j(t,\cdot)$$

$f(\cdot)\otimes g(\cdot)(x) \equiv \int dx_1 dx_2 \delta(x - x_1 x_2) f(x_1) g(x_2)$

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Monte Carlo solution of Evolution Equation

$$\frac{\partial}{\partial t} D_k(t,x) = \sum_j \mathcal{P}_{kj}(t,\cdot) \otimes D_j(t,\cdot)$$

Differential equation $\longrightarrow$ integral equation:

$$e^{\Phi_k(t,t_0)} D_k(t,x) = D_k(t_0,x) + \int_{t_0}^{t} dt_1 e^{-\Phi_k(t_1,t_0)} \sum_j \mathcal{P}^{\Theta}_{kj}(t_1,\cdot) \otimes D_j(t_1,\cdot)(x)$$

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and the Sudakov formfactor appears

$$\Phi_k(t,t_0) = \int_{t_0}^{t} dt'\ \mathcal{P}^{\delta}_{kk}(\epsilon(t'))$$

Iterative multi-integral solution

$$
\begin{aligned}
D_K(t,x) &= e^{-\Phi_K(t,t_0)} D_K(t_0,x) \\
&\times + \sum_{n=1}^{\infty} \sum_{K_0 \dots K_{n-1}} \prod_{i=1}^{n} \left[\int_{t_0}^{t} dt_i \; \Theta(t_i - t_{i-1}) \int_0^1 dz_i \right] \\
&\times e^{-\Phi_K(t,t_n)} \int_0^1 dx_0 \; \prod_{i=1}^{n} \left[\quad \mathcal{P}^{\Theta}_{K_i K_{i-1}}(t_i, z_i) e^{-\Phi_{K_{i-1}}(t_i,t_{i-1})} \right] \\
&\times \quad D_{K_0}(t_0,x_0) \delta\big(x - x_0 \prod_{i=1}^{n} z_i\big),
\end{aligned}
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where $K_n \equiv K$. Many options for the MC implementation.
Generally they can be Markovian OR non-Markovian.

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&\times e^{-\Phi_K(t,t_n)} \int_0^1 dx_0 \; \prod_{i=1}^{n} \left[z_i \mathcal{P}^{\Theta}_{K_i K_{i-1}}(t_i, z_i) e^{-\Phi_{K_{i-1}}(t_i, t_{i-1})} \right] \\
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\end{aligned}
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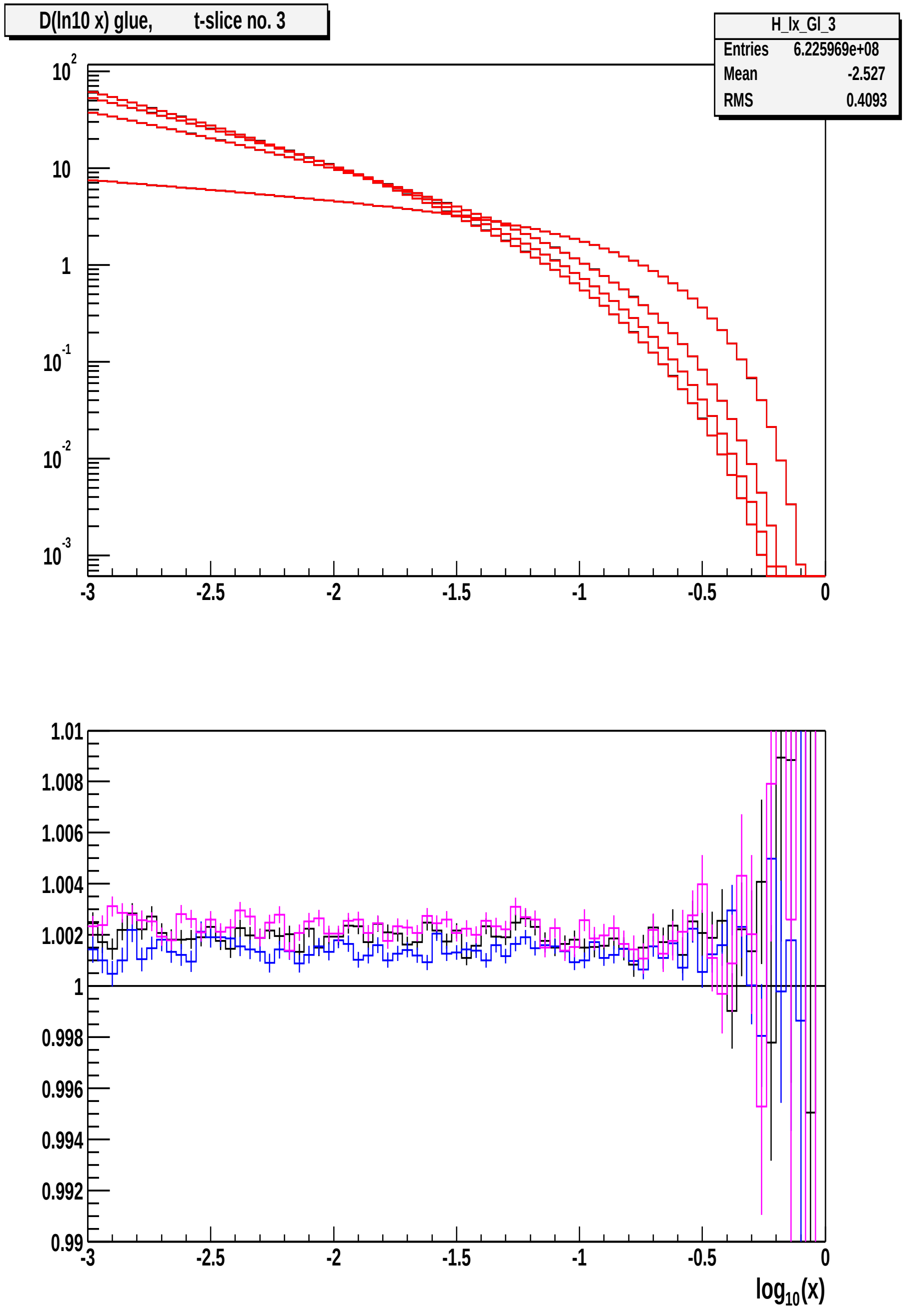
where $K_n \equiv K$. Many options for the MC implementation.
Generally they can be Markovian OR non-Markovian.
Solution for energy parton distributions $xD(x)$ more convenient!

Why? Kernels obey sum rules: $\sum_X \int dz \; z\mathcal{P}_{XK}(z) = 1$.

Master equation for Markovian solution

$$
\begin{aligned}
xD_K(\tau,x) &= \int\limits_{\tau_1>t} d\tau_1 dz_1 \sum_{K_1} \bar{\omega}(\tau_1,x_1,K_1|\tau_0,x_0,K)\;\; xD_K(\tau_0,x) \\
&+ \sum_{n=1}^{\infty} \int_0^1 dx_0 \int\limits_{\tau_{n+1}>\tau} d\tau_{n+1} dz_{n+1} \sum_{K_{n+1}} \sum_{K_0\ldots K_{n-1}} \prod_{i=1}^{n} \int\limits_{\tau_i<\tau}^{t} d\tau_i dz_i \\
&\quad \times \bar{\omega}(\tau_{n+1},x_{n+1},K_{n+1}|\tau_n,x_n,K_n) \qquad \leftarrow \text{spillover} \\
&\quad \times \prod_{i=1}^{n} \bar{\omega}(\tau_i,x_i,K_i|\tau_{i-1},x_{i-1},K_{i-1}) \qquad \leftarrow \text{normal step} \\
&\quad \times \delta\Big(x - x_0 \prod_{i=1}^{n} z_i\Big)\;\; x_0 D_{K_0}(\tau_0,x_0)\; \bar{w}_P\; \bar{w}_\Delta \qquad \leftarrow \text{MCweight}
\end{aligned}
$$

Tests: Proton $\rightarrow$ gluon

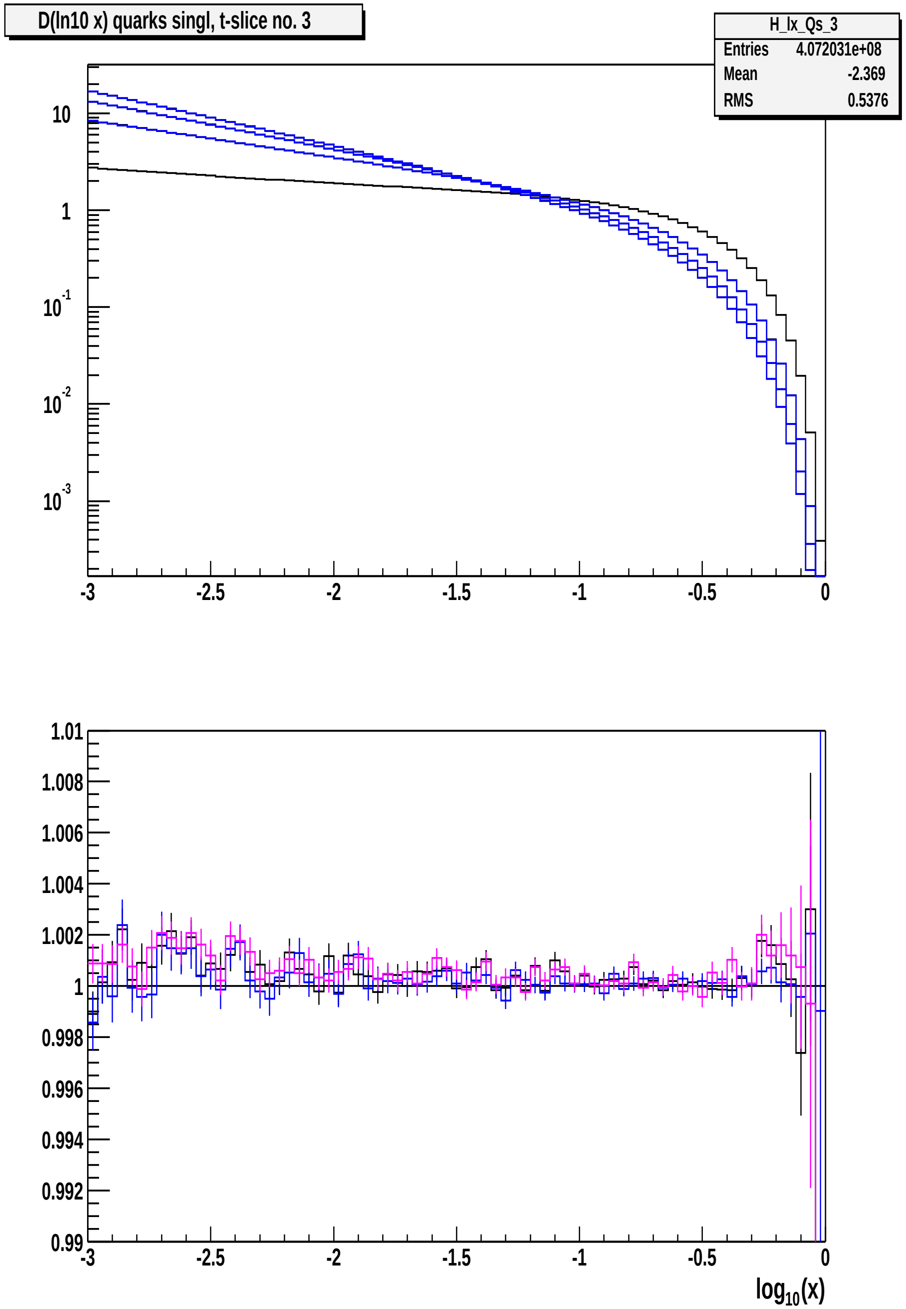


Upper plot shows gluon distribution $xD_G(x, Q_i)$ evolved from $Q_0 = 1$GeV to $Q_i = 10, 100, 100$GeV obtained from `QCDnum16` and `EvolMC1`, while lower plot shows their ratio.

The horizontal axis is $\log_{10}(x)$.

Starting distribution is complete proton at $Q = 1$GeV.

Tests: Proton $\rightarrow$ quarks



Upper plot shows quark singlet distribution $xD_G(x, Q_i)$ evolved from $Q_0 = 1$GeV to $Q_i = 10, 100, 100$GeV obtained from `QCDnum16` and `EvolMC1`, while lower plot shows their ratio.

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Starting distribution is complete proton at $Q = 1$GeV.

Proton composition at 1GeV

This is what we took for the introductory exercise:

$$xD_G(x) = 1.9083594473 \cdot x^{-0.2}(1-x)^{5.0},$$
$$xD_q(x) = 0.5 \cdot xD_{\rm sea}(x) + xD_{2u}(x),$$
$$xD_{\bar{q}}(x) = 0.5 \cdot xD_{\rm sea}(x) + xD_d(x),$$
$$xD_{\rm sea}(x) = 0.6733449216 \cdot x^{-0.2}(1-x)^{7.0},$$
$$xD_{2u}(x) = 2.1875000000 \cdot x^{0.5}(1-x)^{3.0},$$
$$xD_d(x) = 1.2304687500 \cdot x^{0.5}(1-x)^{4.0},$$

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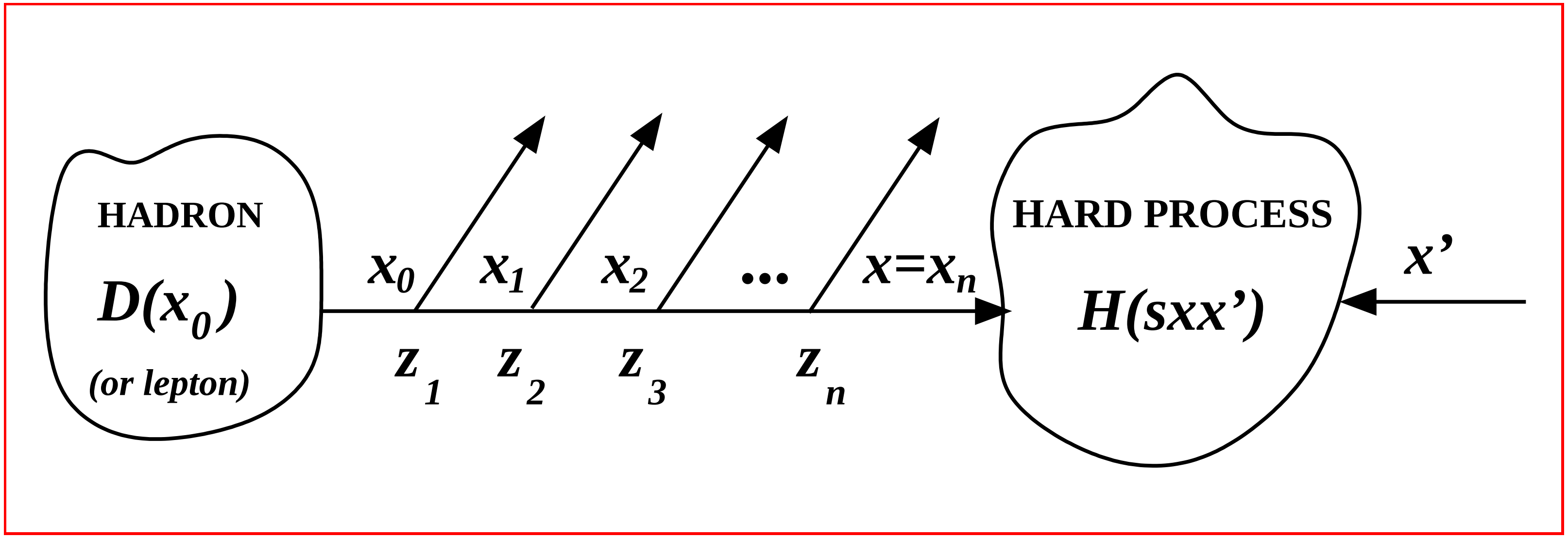
- Introductory exercise: Markovian MC `EvolMC` was found to agree with `QCDnum16` to within 0.2%, Acta Phys.Polon. B35 (2004) 745

- **Recently, 1-st prototype of the efficient *constrained Markovian MC* (solution IIB) prototyped.**

Constrained Solutions class I and II



$$\int dx_0\ D(x_0) \int \prod_i dz_i P(z_i)\ H\big(sx_0 \prod z_i\big)$$

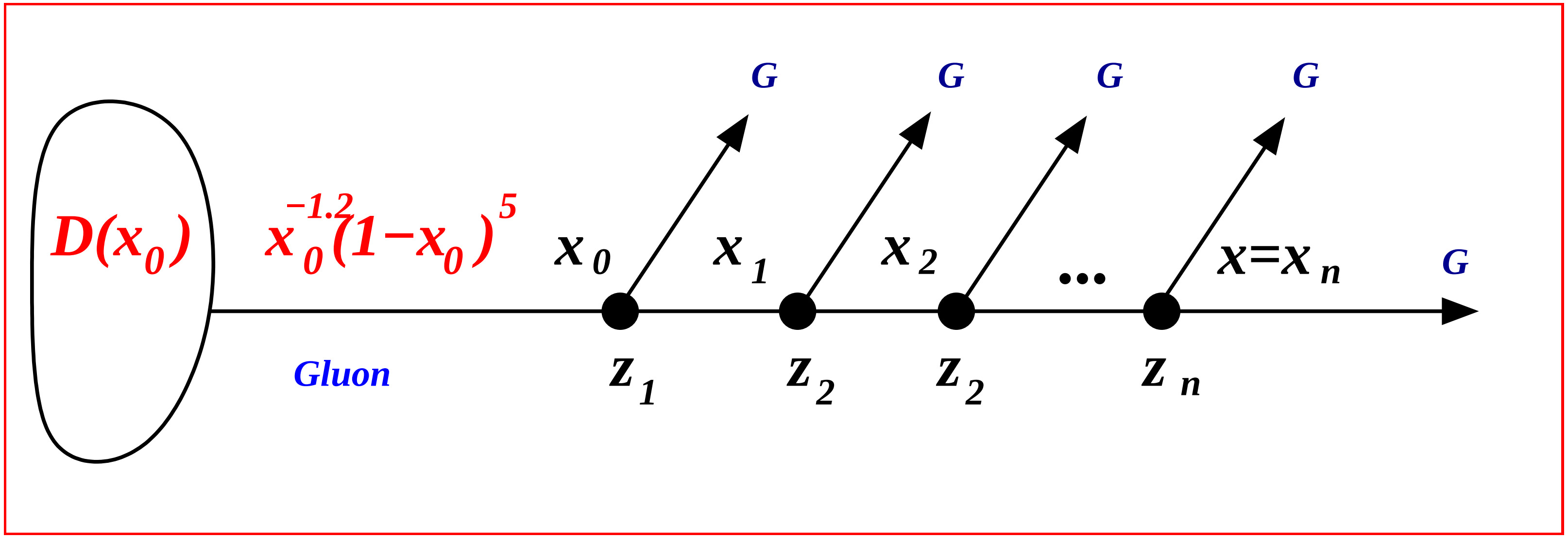
Solutions class I (more difficult because of $\delta(\dots)$):

$$\int dx\ dx_0\ D(x_0) H\big(sx\big) \int \prod_i dz_i P(z_i) \delta(x - x_0 \prod_i z_i)$$

Solutions class II (only for QCD) **NEW!**:

$$\int dx\ H\big(sx\big) \int \prod_i \frac{dz_i}{z_i} P(z_i)\ \ D(x/\prod_i z_i)\ \Theta(\prod z_i - x)$$

Prototype IIB

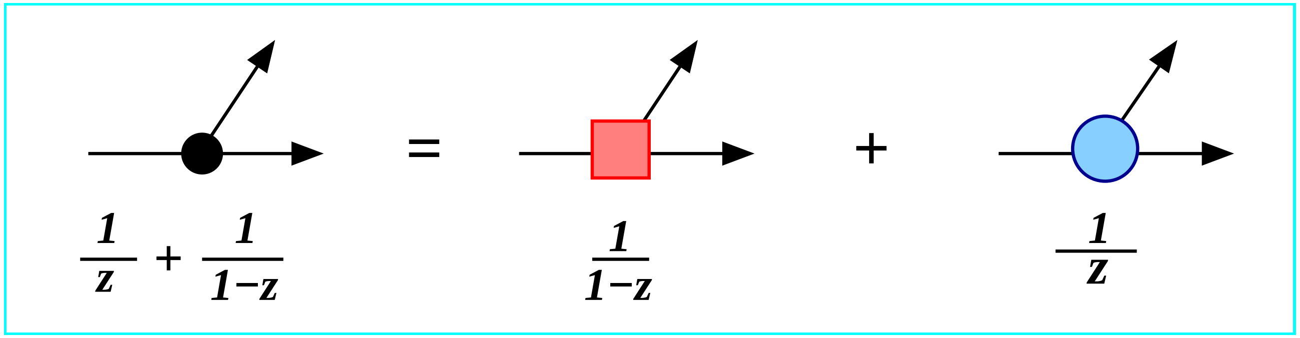


Replace $D(x_0) \to 1/x_0 = x \prod \frac{1}{z_i}$. Compensated by MC weight.

Must generate $P(z_i) = 2C_A\left(\frac{1}{z_i} + \frac{1}{1-z_i}\right)$

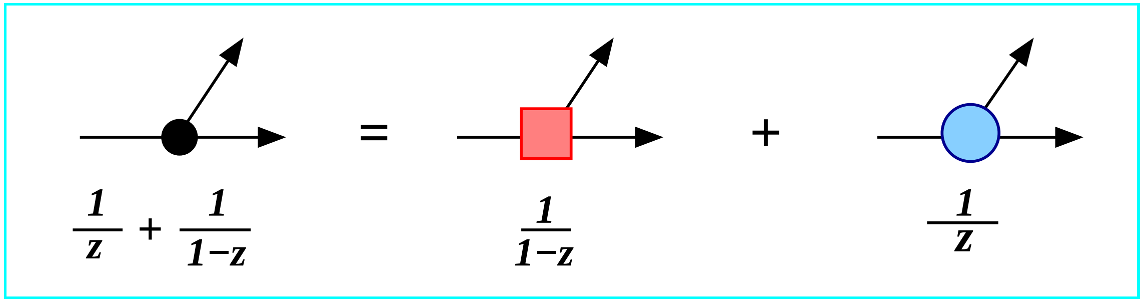
with the constraint $\prod_i z_i \geq x$. Not so trivial!

Solution by the multibranching method:

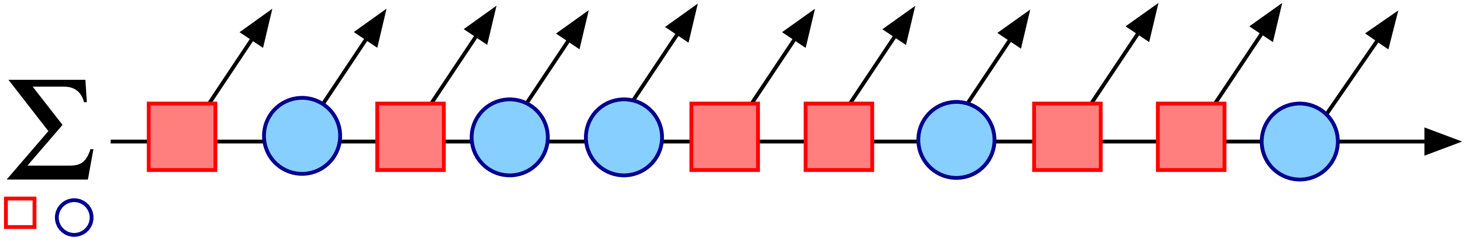


Multibranching in IIB

Using



Leads to sum over branches:



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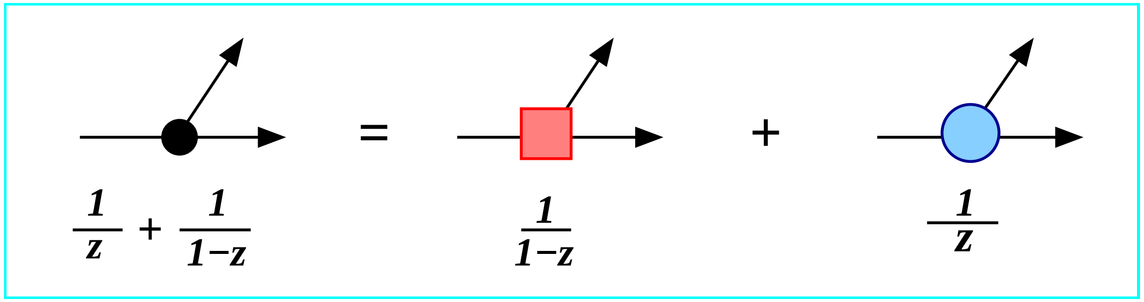

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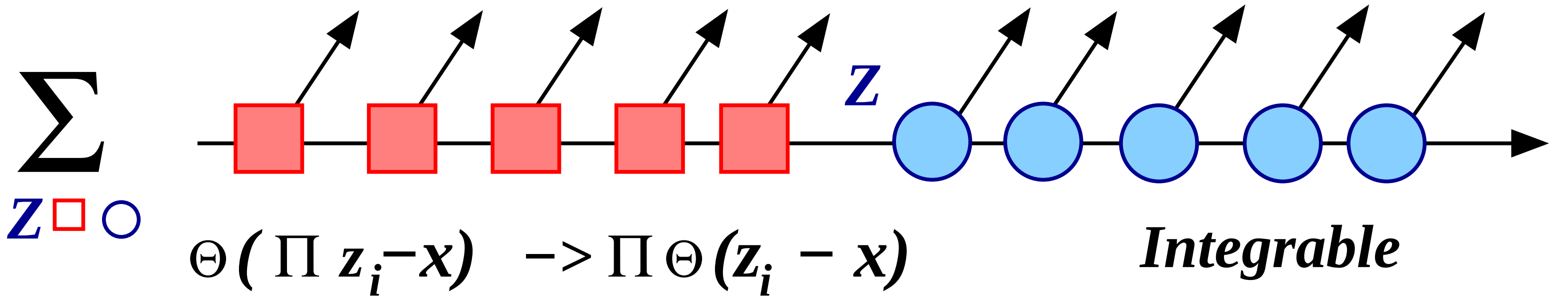

Contributions $1/z$ and $1/(1-z)$ are combined and resummed separately.

Multibranching in IIB

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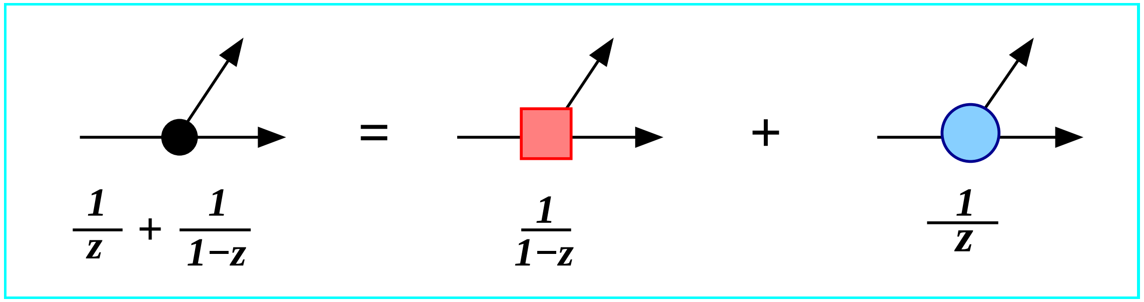

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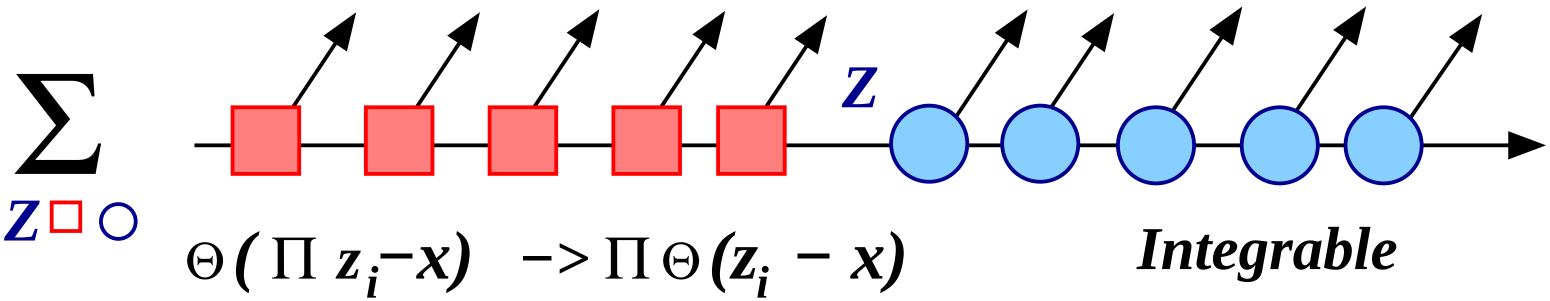

Contributions $1/z$ and $1/(1-z)$ are combined and resummed separately.
Worst-case scenario (pure gluon bremsstrahlung) is now prototyped and tested.

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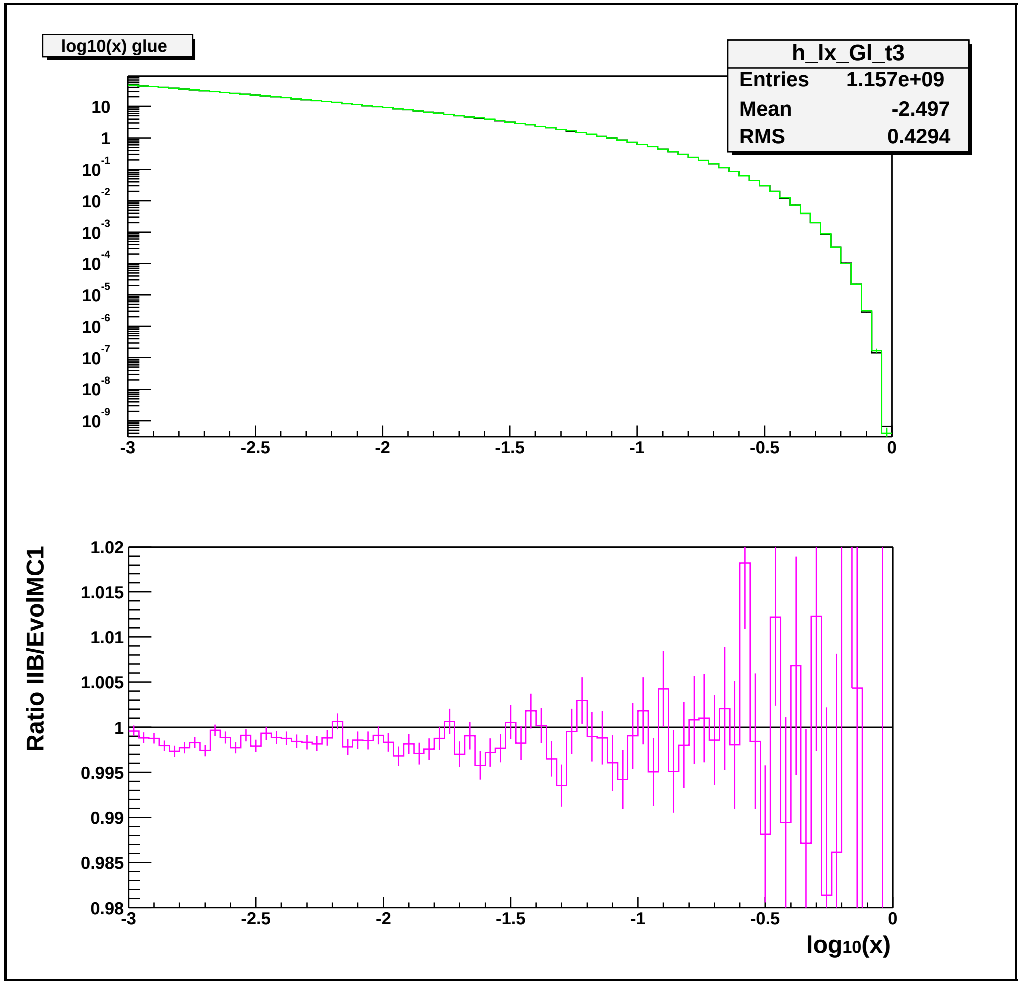
- Introductory exercise: Markovian MC `EvolMC` was found to agree with `QCDnum16` to within 0.2%, Acta Phys.Polon. B35 (2004) 745
- Recently, 1-st prototype of the efficient constrained Markovian MC (solution IIB) prototyped.
- It agrees with the Markovian `EvolMC` to within 0.2%

Testing prototype IIB



Comparison of IIB solution with the Markovian MC `EvolMC` for pure gluonstrahlung. Two solutions and the ratio (lower plot).

Agreement to within 0.2%

Short-term prospects

- More testing of IIB.
- Numerical test of solutions class I
  (several solutions found, under tests)
- Implementing transitions $Q \to G$ and $G \to Q$
  (at least 2 methods found)
- Adding NLL corrections
  (looks rather trivial)

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- Bottom line:
  NEW AVENUES are opened in the construction of the ISR PARTON SHOWER type MCs