Reinventing Parton Shower Monte Carlo

Monte Carlo modelling of NLO DGLAP QCD Evolution

in the fully unintegrated form

S. JADACH

in collaboration with

A. Kusina, M. Skrzypek and M. Sławińska

IFJ-PAN, Kraków, Poland

Partly supported by EU grant MRTN-CT-2006-035505, and Polish Government grant N N202 128937.

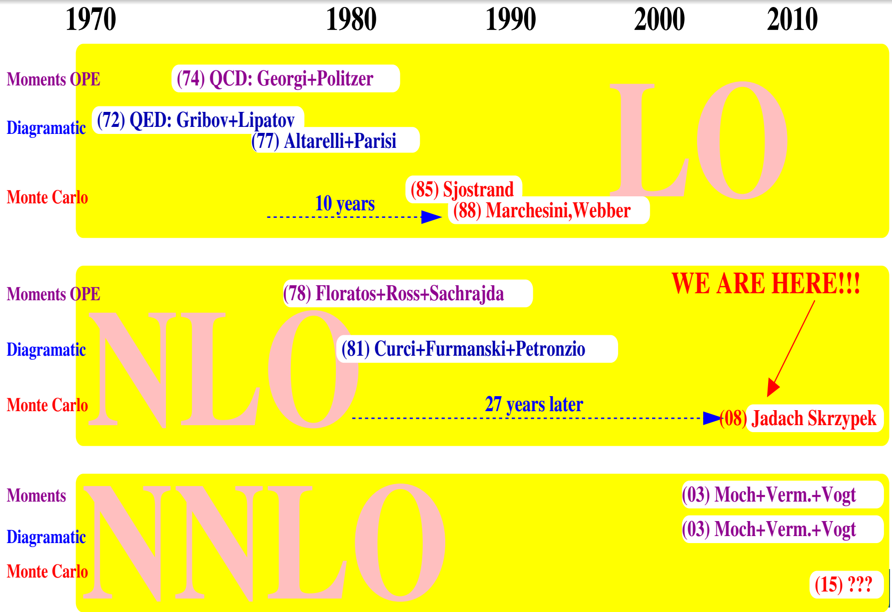
Presented at **50 Cracow School of Theoretical Physics**, Zakopane, June 9-19th, 2010

More on http://jadach.web.cern.ch/

Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form; Mission statement:

Three-decades old paradigm of perturbative QCD calculations states that: hard process matrix element calculated to LO+NLO(+NNLO) level is combined with (i) either the collinear PDF at LO+NLO(+NNLO) or (ii) with the Monte Carlo parton shower, **but the MC PS is restricted to LO only!** For many years upgrading Monte Carlo parton shower to NLO level was regarded as **unfeasible in practice or in principle**, or both. In a series of the recent works we demonstrate, that for NLO non-siglet subset of diagrams we are able to implement in the Monte Carlo PS the NLO DGLAP evolution, without any approximation. This work, after extending to complete NLO DGLAP, will lead to a new class of powerful techniques of combining resummed and finite order pQCD calculations in a form of Monte Carlo event generators, for LHC and beyond.

DGLAP Collinear QCD ISR Evolution in the Monte Carlo



Why 20 years time lag?

Many reasons:

1. Lack of motivation – poor exp. data from hadron colliders
2. QCD Parton shower Monte Carlo (PSMC) main objective was (still is?) hadronization
   – until recently not involved in pQCD @ hard process
3. Conceptual barriers: Factorisation theorems (EGMPR, CFP, CSS, Bodwin,...) not suited for MC beyond LO:
   - non-conservation of 4-momenta
   - over-subtractions $\rightarrow$ huge cancellations
   - non-positive distributions
   - real emissions irreversibly integrated over
4. CPU time: in 1985 No. of MC events <50k, $\sim$ 3% precision; now 2010, $2 \cdot 10^{10}$ events and $\sim$ 0.01% errors are routine.

Our expection: NLO PS era in QCD MC is coming!
After completing NLO and NNLO calculations for hard processes NLO PSMC may/will become the main front of pQCD activity!

Possible profits/gains from NLO PSMC

- Complete set of “unitegrated soft counterterms” for combining hard process ME at NNLO with NLO PS MC
- Natural extensions towards BFKL/CCFM at low x
- Better modelling of low scale phenomena, $Q < 10 GeV$, quark thresholds, primordial k^T, underlying event, etc.
- Porting information on parton distributions from DIS (HERA) to W/Z/DY (LHC) in the MC itself, instead in form of collinear PDFs (universality must be preserved)
- and more...

MC modelling of NLO DGLAP is not the aim in itself – it will be a starting platform for many developments in many directions.

The aim of the present exercise (KRKMC project)

Constructing NLO Parton Shower Monte Carlo for QCD Initial State Radiation for one initial parton:

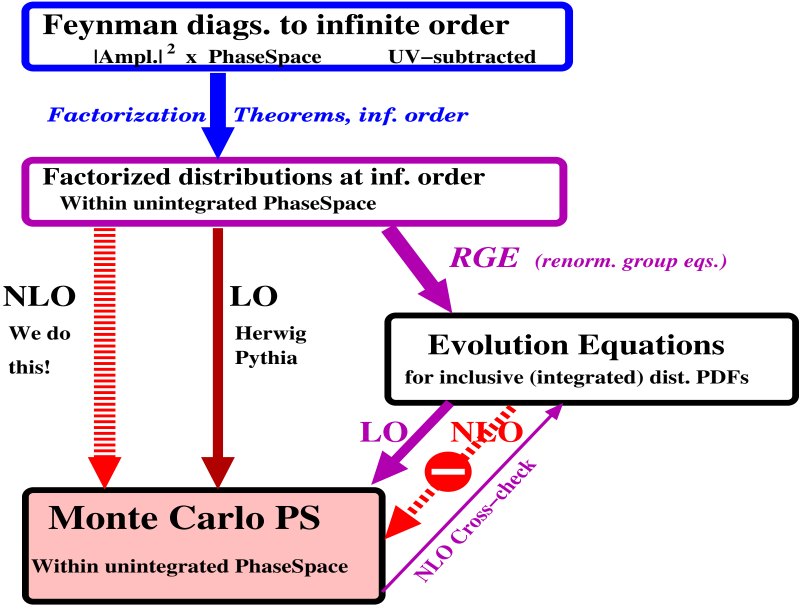
- based on the collinear factorisation (EGMPR, CFP, CSS, Bodwin...) as rigorously as we can,
- CFP=Curci-Furmanski-Petronzio scheme as a main reference/guide (axial gauge, $\overline{MS}$ dim. regulariz.),
- implementing *exactly* NLO DGLAP evolution,
- and fully unintegrated exclusive PDFs (ePDFs),
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels

We are going to show that it is feasible
– the proof of the concept for non-singlet NLO DGLAP.

More details on the project are available here:

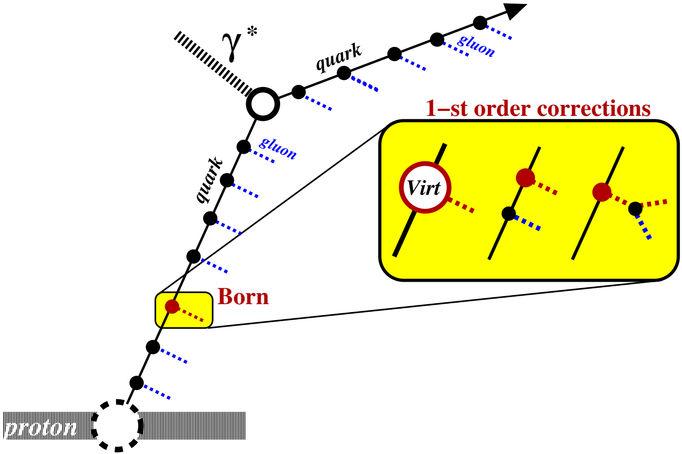
- Epiphany 2009 Proceedings,
  article http://arxiv.org/abs/0905.1399
  slides http://home.cern.ch/jadach/public/epip09.pdf
  More on re-inserting NLO corrections into LO MC ($\sim C_F^2$)...
- RADCOR 2009 Proceedings,
  article http://arxiv.org/abs/1002.0010
  slides http://home.cern.ch/jadach/public/RADCOR09.pdf
  More on the “factorization scheme” in our NLO PSMC versus CFP/EGMPR...

QCD factorsation versus Monte Carlo - An overview



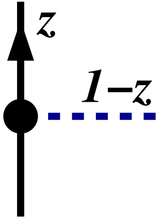
Leading Order (LO) ladder vertex is our "Born"
Emission of gluons out of quark

The aim is to implement in the Monte Carlo complete NLO DGLAP in the initial state ladder, using unintegrated Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge).

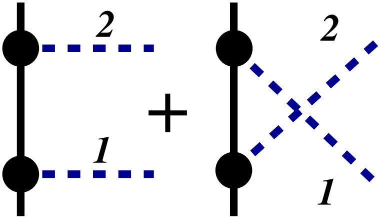


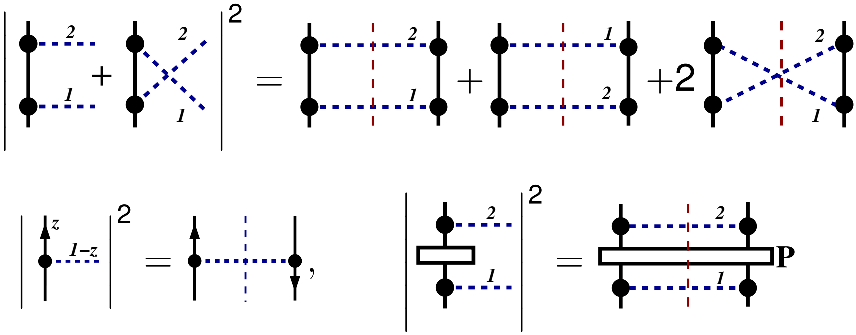
1-st order virtual and real correction (subset) diagrams

Virtual : $\left(1 + \Delta^{(1)}_{ISR}(z)\right)$



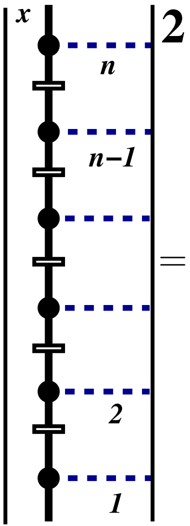
Real :




2
1
+
2
1
2
=
2
1
+
1
2
+2
2
1
z
1−z
2
=
,
2
1
2
=
2
P
1

LO ladder = parton shower MC

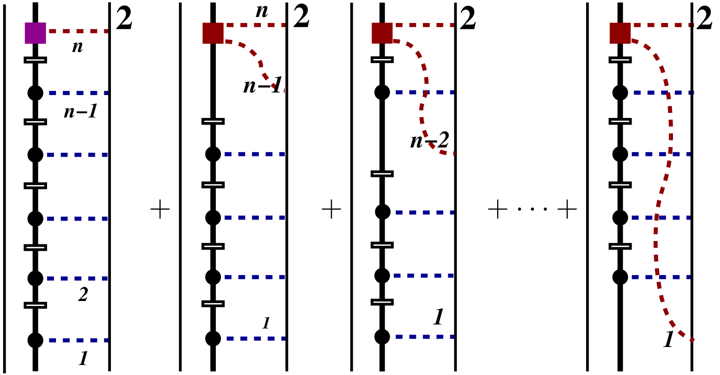
$$\sum_{n=0}^{\infty}$$



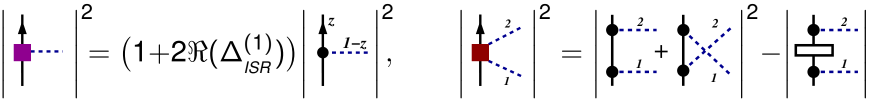
$$= e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0}\, \theta_{Q>a_i>a_{i-1}}\, \rho_{1B}^{(0)}(k_i)\, \delta_{x=\prod z_i}$$

$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}.$$

LO with NLO-corrected kernel at the end of the ladder

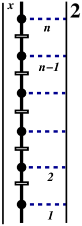


Virt. multiplicative   Undoing LO simplificat.   Sum over trailing LO spectators, essential (BE, YFS61)

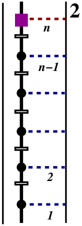


MORE DETAILS:

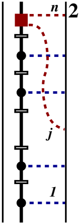
$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}}$



$+\, e^{-S_{ISR}}$



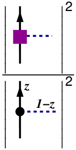
$+\, e^{-S_{ISR}} \sum_{j=1}^{n-1}$



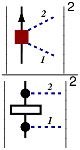
$= e^{-S_{ISR}} \Big\{ \delta_{x=1} +$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Big\},$$

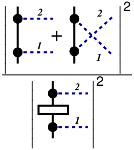
where $d\eta_i = \frac{d^3 k_i}{k_i^0}$, $\quad \beta_0^{(1)} =$



$,\quad W(k_2,k_1) =$



$=$



$- 1.$

Mapping $k_i \to \tilde{k}_i$ instrumental. S_{ISR} = double-log Sudakov, W is non-singular!

Algebraic crosscheck

Analytical integration of NLO part $\sum_j W(\tilde{k}_n, \tilde{k}_j)$ can be done leading to:

$$\sum_{n=1}^{\infty} \int du \int\limits_{Q>a_n>a_{n-1}} \frac{da_n}{a_n} \mathcal{P}^{(1)}_{qq}(u) \left(\prod_{i=1}^{n-1} \int\limits_{a_{i+1}>a_i>a_{i-1}} \frac{da_i}{a_i} \mathcal{P}^{(0)}_{qq}(z_i) \right) \delta_{x=u\prod_{j=1}^{n-1} z_j}.$$

where we recover precisely NLO part (including virtuals) of standard DGLAP kernel $\mathcal{P}^{(1)}_{qq}(u)$ defined according to:

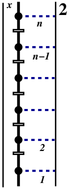
$$\mathcal{P}^{(1)}_{qq}(u) \ln\frac{Q}{q_0} = \int\limits_{Q>a_n>a_0} d^3\eta_n\, \rho^{(1)}_{1B}(k_n)\, \beta^{(1)}_0(z_n)\delta_{u=z_n} + \int\limits_{Q>a_n>a_0} d^3\eta_n \int\limits_{a_n>a_{n'}>0} d^3\eta_{n'}\, \beta^{(1)}_1(\tilde{k}_n, \tilde{k}_{n'})\, \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced.

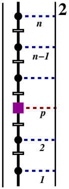
NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p:

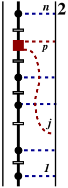
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p=1}^{n}$$



$$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$



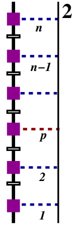
$$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

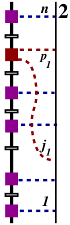
Next step is to add more "NLO insertions",
for instance 2 at positions p_1 and p_2 and sum up over them...
then 3 insertions at p_1, p_2, p_3 and so on
– LO+NLO kernels built up all over along the ladder!

NLO-corrected kernels all over the ladder, $\sim C_F^2$

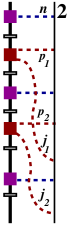
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{ \Bigg|$$



$$\Bigg|^2 + \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1} \Bigg|$$



$$\Bigg|^2 + \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1\\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1\\ j_2\neq p_1,j_2}}^{p_2-1} \Bigg|$$



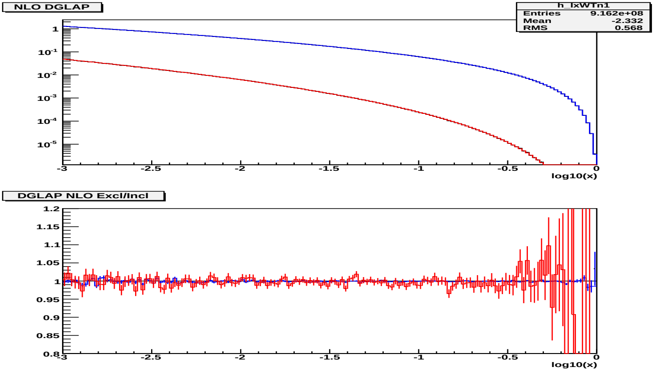
$$\Bigg|^2 \Bigg\}$$

$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \Bigg) \Bigg[1 + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) +$$

$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1\\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1\\ j_2\neq p_1,j_2}}^{p_2-1} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \dots \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.

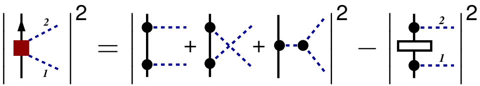
Numerical test of ISR pure C_F^2 NLO MC



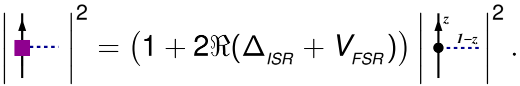
Numerical results for $D(x, Q)$ from inclusive and exclusive **two** Monte Carlos.
Blue curve is single NLO insertion, red curve is double insertion component.
LO+NLO is off scale. Evolution 10GeV→1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

THE PROBLEM WITH GLUON PAIR COMPONENT OF the NLO KERNEL, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:



and $-S_{FSR}$ in the virtual correction:

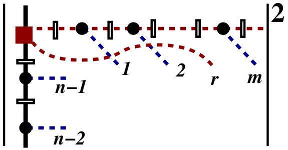


SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

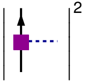
NLO FSR corr. at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR corr. at the end of the ladder:

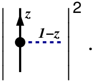
$$e^{-S_{ISR}-S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$



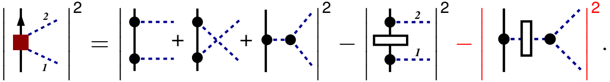
where Sudakov S_{FSR} is subtracted in the virtual part:



$$= \left(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\right)$$



and FSR counterterm is subtracted in the 2-real-gluon part:



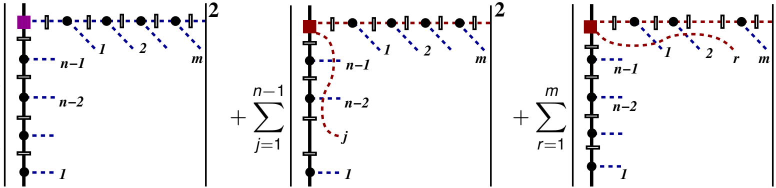
The miracle: both are free of any collinear or soft divergency!!!

The most important point in this talk, next slide:

ISR+FSR NLO scheme, NLO corr. at end of the ladder

$$\bar{D}^{[1]}_{NS}(x,Q) =$$

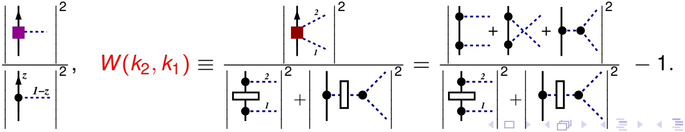
$$e^{-S} \sum_{n,m=0}^{\infty} \Big\{ \ldots + \sum_{j=1}^{n-1} \ldots + \sum_{r=1}^{m} \ldots \Big\}$$



$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho^{(1)}_{1B}(k_i) \right) e^{-S_{FSR}} \sum_{m=0}^{\infty} \left(\prod_{j=1}^{m} \int_{Q>a_{nj}>a_{n(l-1)}} d^3\eta'_j \; \rho^{(1)}_{1V}(k'_j) \right)$$

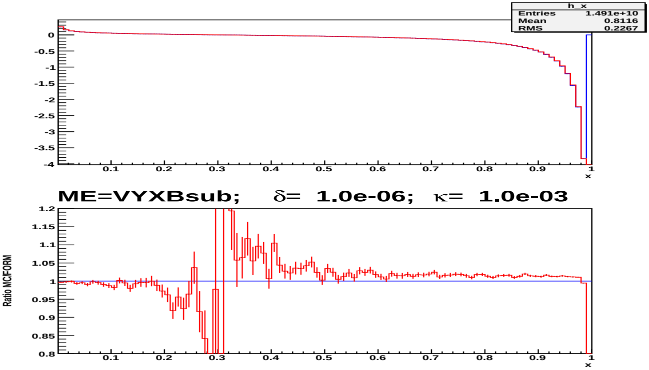
$$\times \left[\beta^{(1)}_0(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^{m} W(\tilde{k}_n, \tilde{k}'_r) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

$$\beta^{(1)}_0 \equiv \ldots, \quad W(k_2,k_1) \equiv \ldots = \ldots - 1.$$



3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:

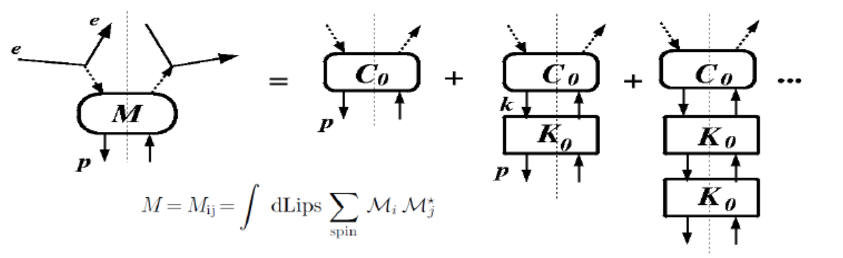


because in this case analytical integration is feasible.
MC agrees precisely with the analytical result.

A litle bit more on theory of the collinear factorization in a form suitable for the Monte Carlo

EGMPR scheme of collinear factorization (1978)

"Raw" factorization of the IR collinear singularities



- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0(1 + K_0 + K_0^2 + \cdots) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

EGMPR scheme customized to $\overline{MS}$ by Furmanski and Petronzio (80):

$$F = C_0 \cdot \frac{1}{1-K_0} = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right),$$

$$= \left\{ C_0 \cdot \frac{1}{1-(1-\mathbb{P}) \cdot K_0} \right\} \otimes \left\{ \frac{1}{1-\left(\mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P}) \cdot K_0}\right)} \right\}_{\otimes},$$

$$\Gamma\left(\alpha, \frac{1}{\epsilon}\right) \equiv \left(\frac{1}{1-K}\right)_{\otimes} = 1 + K + K \otimes K + K \otimes K \otimes K + ...,$$

$$K = \mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P}) \cdot K_0}, \quad C = C_0 \cdot \frac{1}{1-(1-\mathbb{P}) \cdot K_0}.$$

Ladder part Γ corresponds to MC parton shower
C is the hard process part
$\mathbb{P}$ is the projection operator: $\mathbb{P} = P_{spin}\, P_{kin}\, PP$

For MC we use right now brute force interpretation of collinear ε-poles:

$$\frac{1}{\varepsilon} = \int_0^{\mu_F} \frac{dk^T}{k^T} \left(\frac{k^T}{\mu_F}\right)^{\varepsilon}.$$

CFP (1980) factorization scheme

$$F = C_0 \cdot \frac{1}{1-K_0} = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0} \otimes \Gamma, \quad \Gamma = \frac{1}{1-\left(\mathbb{P}K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)},$$

introduces enormous oversubtractions/cancellations. At LO we have:

$$\Gamma \simeq \frac{1}{1-\left(1-e^{-\frac{1}{\varepsilon}}\right)} = 1 + \left(1-e^{-\frac{1}{\varepsilon}}\right) + \left(1-e^{-\frac{1}{\varepsilon}}\right)^2 + \ldots$$

while from RGE and explicit LO calculation give us directly

$$\Gamma = e^{+\frac{1}{\varepsilon}} = 1 + \frac{1}{\varepsilon} + \frac{1}{2!}\frac{1}{\varepsilon^2} + \ldots$$

We want this exponent directly from the Feynman diagrams!!!

This is what we actually implement in the present MC!

$$F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^s K_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu)$$

$$\overleftarrow{\mathbb{R}}_\mu(K_0) = \overleftarrow{\mathbb{B}}_\mu\left[\frac{1}{1-K_0}\right] \equiv 1 + \overleftarrow{\mathbb{B}}_\mu[K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] + \dots$$

Explaining the notation/meaning step by step:

- $\exp_{TO}$ means time ordered exponential in the time evolution variable = log of factorization scale, next slide.
- Operator $\overleftarrow{\mathbb{B}}$ is defined recursively (similarly as β-functions in Yennie-Frautschi-Suura 1961 subtraction scheme):

$$\overleftarrow{\mathbb{B}}_\mu[K_0] = K_0 - \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] = K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \mathbb{P}'_\mu\{{}^{s_2}K_0 \cdot \overleftarrow{\mathbb{B}}_{s_2}[K_0]\} - \overleftarrow{\mathbb{B}}_\mu[K_0] \cdot \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] = K_0 \cdot K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_3}K_0\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \dots$$

- The key point is the definition of new $\mathbb{P}'$ projection operator.

New factorization formula = algebraic structure for MC

$$F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu)$$

Modified projection operator $\overleftarrow{\mathbb{P}}'$:

- Does spin projection as in $\mathbb{P}$ of CFP
- sets on-shell all (cut) real momenta towards hard process
- acts on integrand, leaves intact Lorentz invar.ph.sp. (LIPS)
- sets upper limit μ on the phase space for all real (cut) partons towards hardon eg. $\mu > s(k_1, .., k_n) = \max(k_i^T)$,
- our preferred choice is rapidity ordering choice: $s(k_1, .., k_n) = a(k_1, .., k_n) = \max(k_i^T/\alpha_i)$, $\alpha_i = k_i^+/E$
- $\overleftarrow{\mathbb{P}}(A)$ acts on A which is at most single-log (col.) divergent and extracts this singularity from the LIPS integrand, (for instance by rescaling all $k_i^T \to \lambda k_i^T$ and taking coefficient in front of $1/\lambda$). NB. $\overleftarrow{\mathbb{P}}'(K_0)$ is OK. because K_0 is single-log divergent.
- Nesting like $\overleftarrow{\mathbb{P}}[K_0 \cdot (1 - \overleftarrow{\mathbb{P}}(K_0))]$ is allowed, as long as its argument is at most single-log divergent.

$$F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^s K_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu)$$

Time ordered exponential:

$$\exp_{TO}\left(\mathbb{P}'_\mu\{A\}\right)(\mu) = 1 + \mathbb{P}'_\mu\{A\} + \mathbb{P}'_\mu\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\} + \mathbb{P}'_\mu\{{}^{s_3}A\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\} + \dots$$

NOTATION: For $A = \int dLips(k_1, k_2, ..., k_n)\ f(k_1, ..., k_n)$,
where k_i are on-shell cut lines (real emitted partons)
the notation $\{{}^{s_3}A\}$ defines $s_3 = a(a_1, ..., a_n) = \max(a_1, ..., a_n)$.

Hence, term like

$$\mathbb{P}'_\mu\{{}^{s_3}A\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\}$$

has its entire integrand multiplied by $\theta_{\mu > s_3 > s_2 > s_1}$,
where μ is constant and s_i are integration variables dependent.

Evolution and kernels...

In our factoriz. formula $F(Q) = C(Q,\mu) \cdot D(\mu)$, hard process part is $C(Q,\mu) = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0]$. and exclusive PDF (ePDF) is the integrand in:

$$D(\mu) = \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu) = \exp_{TO}(K).$$

LO and NLO truncations of the exclusive evolution kernel K_μ are:

$$K_\mu^{LO} = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0\right\}, \quad \text{taken at } \mathcal{O}(\alpha^1),$$

$$K_\mu^{NLO} = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0 + K_0 \cdot (1 - \overleftarrow{\mathbb{P}}') \cdot K_0\right\}, \text{ truncated at } \mathcal{O}(\alpha^2).$$

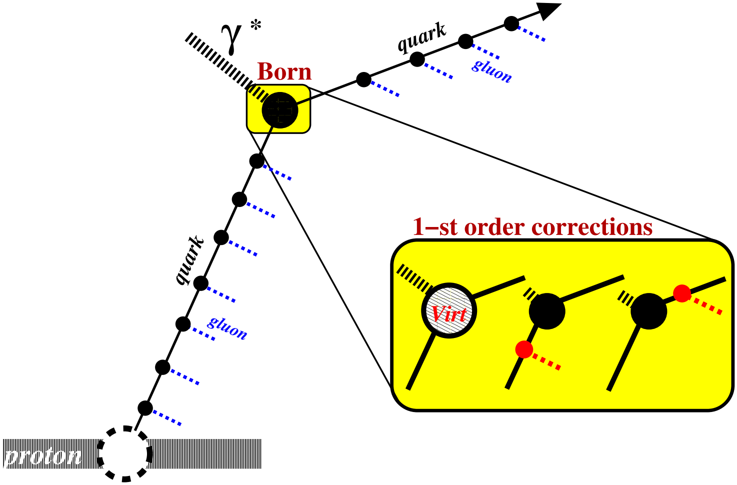
The x-dependent $D(\mu,x)$ obeys ordinary evolution equation:

$$\boxed{\partial_\mu D(\mu,x) = \mathcal{P} \otimes D(\mu)(x)}$$

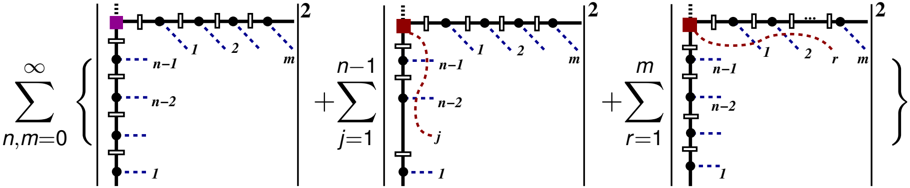
with the traditional inclusive DGLAP kernel being

$$\mathcal{P}(x) = \int d\text{Lips}\, \delta\left(x = \frac{\sum k_i^+}{E_0}\right) \delta\left(1 - \frac{s}{\mu}\right) \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}.$$

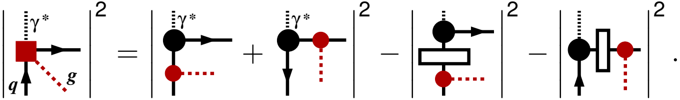
What next? NLO corrections to HARD process M.E.



General scheme of NLO corrections to HARD process



where the following NLO real/virtual corrections/distributions



are free of any double or single collinear/soft singularities.

Summary and Prospects

- First serious feasibility study of the true NLO exclusive parton shower MC is (almost) complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet, including hard process.
- Middle range aim: Complete singlet (Q-G transitions).
- Optimise MC weight evaluation (CPU time).
- Adding NLO hard process into the game (similar to MC@NLO but different).
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Extensions towards CCFM/BFKL, quark masses, fitting PDFs with Monte Carlo.