Matching NLO QCD with parton shower in Monte Carlo scheme - the KrkNLO method

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(based on arXiv:1503.06849)

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Outline and motivation

A method for NLO+PS matching applied to Drell-Yan process,
an alternative to MC@NLO and POWHEG will be presented.

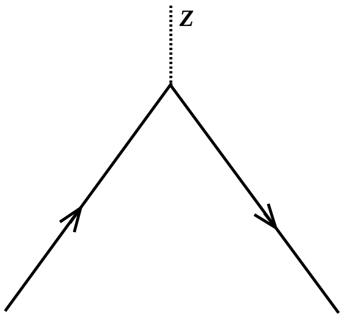
Key ingredients are:

- new factorization scheme leading to new MC PDFs
- NLO correction applied to PS via reweighting of MC events

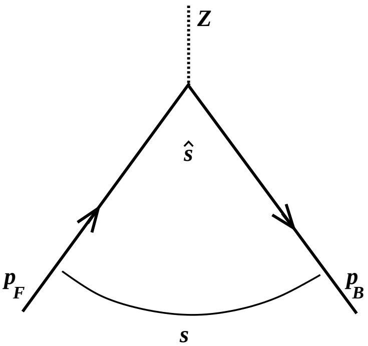
Main differences from established methods, POWHEG and MC@NLO:

- Thanks to departing from $\overline{\text{MS}}$, the NLO+PS matching methos becomes very simple
  $\rightarrow$ single multiplicateve positive MC weight on top of LO PS MC.
- In case of angular ordering in PS MC
  $\rightarrow$ no need for k_T-ordered or truncated showers.
- If it is so simple at NLO+LO PS, one may hope that pushing this method to NNLO hard proc. $\otimes$ NLO PS MC will be feasible.
- It was already checked (beyond scope of this presentation) that the same method works to upgrade PS MC from LO to NLO level.

Drell-Yan process

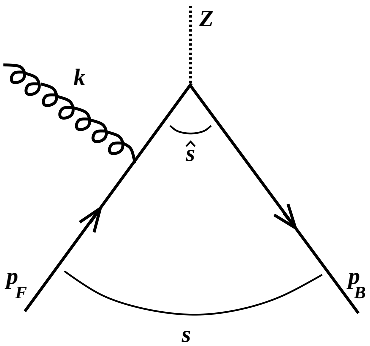


Drell-Yan process



$$s = (p_F + p_B)^2$$

Drell-Yan process



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$$z = \frac{\hat{s}}{s}$$

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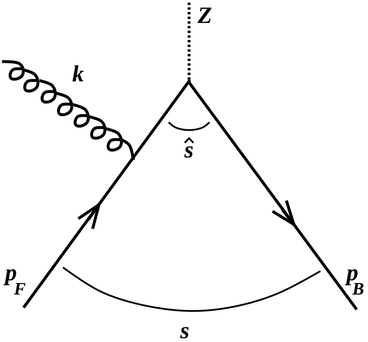

$$s = (p_F + p_B)^2$$

$$z = \frac{\hat{s}}{s}$$

Sudakov variables:

$$\alpha = \frac{2k \cdot p_B}{\sqrt{s}} = \frac{2k^+}{\sqrt{s}}$$

$$\beta = \frac{2k \cdot p_F}{\sqrt{s}} = \frac{2k^-}{\sqrt{s}}$$

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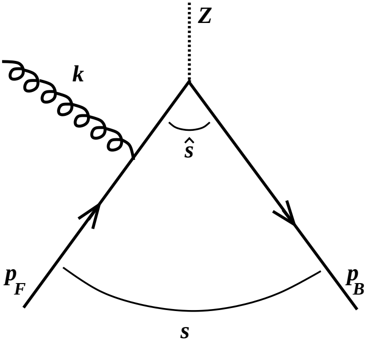

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$$z = 1 - \alpha - \beta$$

$$k_T^2 = s\alpha\beta$$

$$y = \frac{1}{2} \ln \frac{\alpha}{\beta}$$

DY cross section at NLO in collinear $\overline{\text{MS}}$ factorization

For the $q\bar{q}$ channel:

$$d\sigma^1_{\text{DY}} = \sigma^B_{\text{DY}}\, D_1^{\overline{\text{MS}}}(x_1,\hat{s}) \otimes \frac{\alpha_s}{2\pi} C_{2q}^{\overline{\text{MS}}}(z) \otimes D_2^{\overline{\text{MS}}}(x_2,\hat{s})\,,$$

where

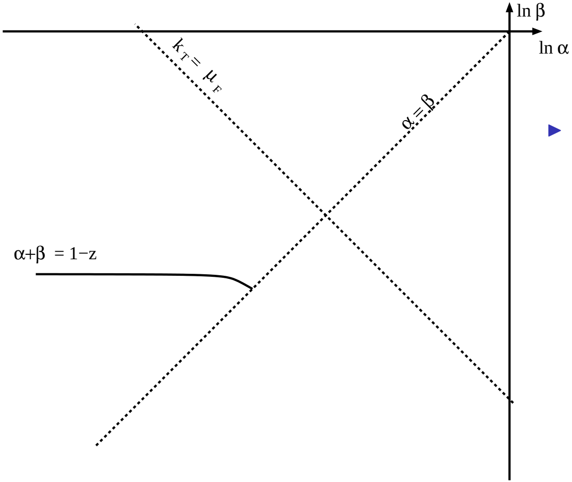
$$C_{2q}^{\overline{\text{MS}}}(z) = C_F \left[4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{1+z^2}{1-z}\ln z + \delta(1-z)\left(\frac{2}{3}\pi^2 - 8\right) \right].$$

We want to reproduce this with Monte Carlo, in a fully exclusive way.

If we decide to use $\overline{\text{MS}}$ PDFs, we need to generate terms of the type $4\,(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+$ that are technical artefacts of $\overline{\text{MS}}$ scheme.

This is problematic since those terms correspond to the collinear limit but Monte Carlo lives in physical 4 dimensions and not in the artificial phase space restricted by $\delta(k_T^2)$.

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



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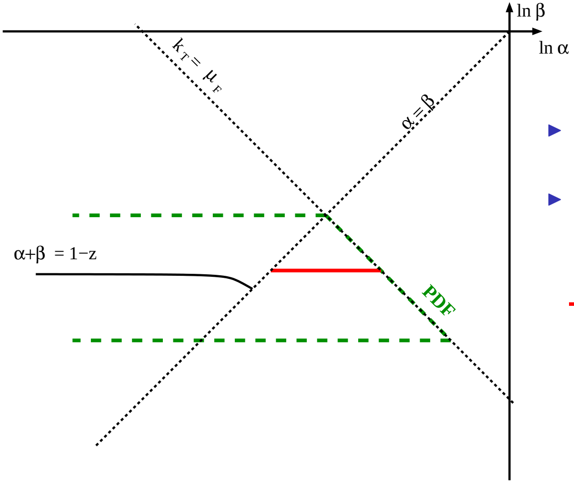

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$$\int \frac{1}{1-z}\frac{d\beta}{\beta} = 2\frac{\ln(1-z)}{1-z}$$

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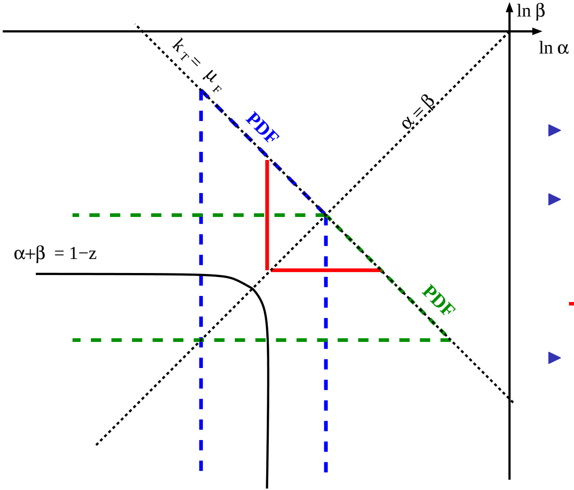

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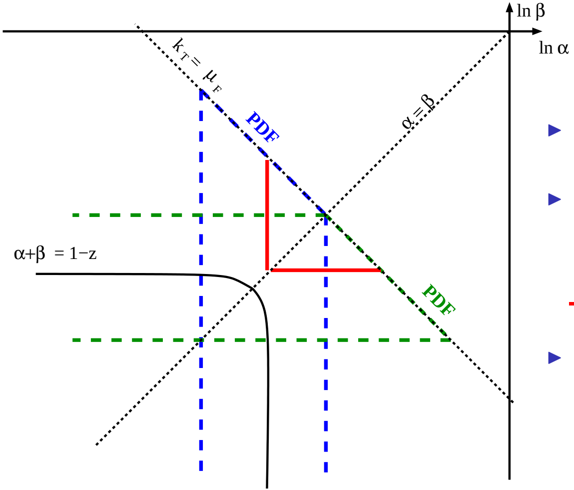

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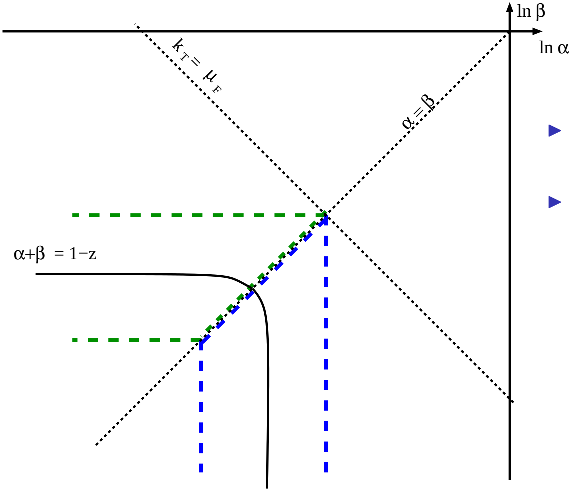
Could we reorganize phase space integration to remove the oversubtraction?

Alternative MC factorization scheme in pictorial way



- Integration in angle rather than k_T.
- No overcounting.

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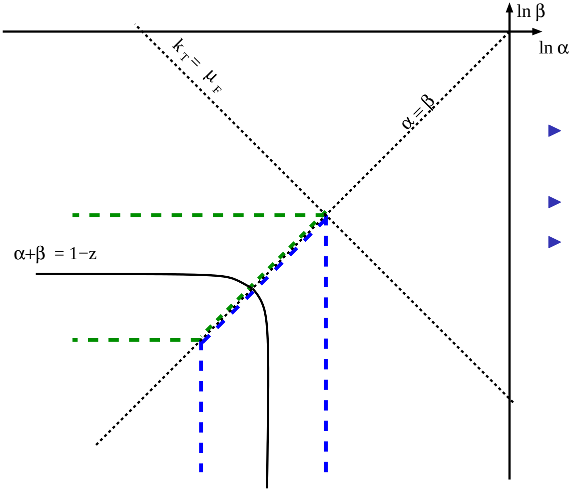

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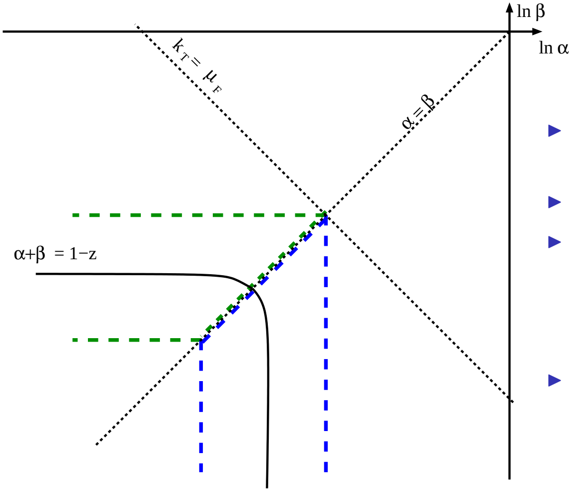

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- Such PDFs respect physical boundaries and correspond directly to MC showers.

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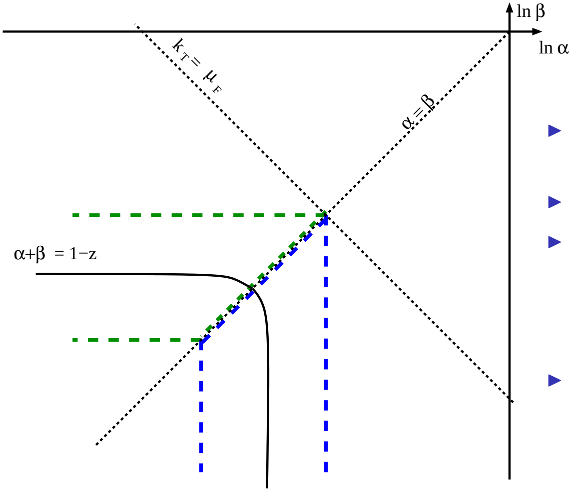

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The change of factorization scheme help us to simplify NLO+PS matching

The KrkNLO method

Two essential steps

1. **Change the factorization scheme from $\overline{\text{MS}}$ to MC**
   - produce new MC PDFs
   - differences at LO
   - universality: recovering $\overline{\text{MS}}$ NLO result

2. **Reweight parton shower**
   - correct hardest emission by 'real' weight
   - upgrade the cross section/distributions to NLO by multiplicative, constant 'soft+virtual' weight

The KrkNLO method

[Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

↪ Developed and validated within a toy model with angular ordering.

- Take a parton shower that covers the (α, β) phase space completely and produces emissions according to approx. matrix element $K \simeq R$.
- Upgrade the real emissions to exact ME by reweighting with R/K.
- Upon integration over transverse d.o.f. this upgraded PS will give an extra contribution $C_2(z) = \int (R - K)$ to the cross section.
- Redefine PDFs by subtracting the above $C_2(z)$ together with all the z-dependent terms from $\overline{\text{MS}}$ coefficient function. This means transforming PDFs to a new MC factorization scheme.
- Virtual+soft correction, Δ_{S+V}, is just a constant now. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

Implementation on top of the Catani-Seymour shower

↪ We used Sherpa 2.0.0 implementation of the CS shower.

Phase space measure of emitted gluon

$$\frac{d\alpha}{\alpha}\frac{d\beta}{\beta} = \frac{d\alpha d\beta}{\beta(\alpha+\beta)} + \frac{d\alpha d\beta}{\alpha(\alpha+\beta)}$$

- The evolution variable:

$$q_F^2 = s(\alpha+\beta)\beta, \qquad q_B^2 = s(\alpha+\beta)\alpha\,,$$

hence

$$\frac{d\alpha d\beta}{\alpha\beta} = \frac{dq_F^2}{q_F^2}\frac{dz}{1-z} + \frac{dq_B^2}{q_B^2}\frac{dz}{1-z}.$$

- The CS shower covers all space of (α, β).

$$\alpha+\beta \leq 1 \quad \Rightarrow \quad z \geq 0 \quad \text{and} \quad q_{F,B}^2 \leq s$$

$$\alpha, \beta > 0 \quad \Rightarrow \quad (1-z)^2 > q_F^2/s \quad \text{or} \quad (1-z)^2 > q_B^2/s$$

Implementation on top of the Catani-Seymour shower

$\hookrightarrow$ It turns out that coefficient functions of the CS shower equal to those from the MC scheme of Jadach et al. arXiv:1103.5015. Hence, CS $\equiv$ MC.

The $C_2(z)$ function:

$$C_2^{\text{MC}}(z)\Big|_{\text{real}} = \int (R - K)$$

- For the $q\bar{q}$ channel:

$$C_{2q}^{\text{MC}}(z)\Big|_{\text{real}} = \frac{\alpha_s}{2\pi} C_F \left[-2(1-z)\right]$$

- For the qg channel:

$$C_{2g}^{\text{MC}}(z)\Big|_{\text{real}} = \frac{\alpha_s}{2\pi}\, T_R\, \frac{1}{2}(1-z)(1+3z)$$

- Quark and anti-quark PDFs are redefined by:
  - subtracting $C_{2q}^{\text{MC}}(z)$ and $C_{2g}^{\text{MC}}(z)$ from $\overline{\text{MS}}$ PDFs
  - absorbing all z-dependent terms from $\overline{\text{MS}}$ coefficient functions
- The virtual correction:

$$C_{2q}\Big|_{\text{virt}} = \delta(1-z)\left(\frac{4}{3}\pi^2 - \frac{5}{2}\right)$$

is applied multiplicatively.

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Simple form of the coefficient functions with no singular terms!

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MC PDFs

Transformation from $\overline{\text{MS}}$ to MC. The (anti-)quark PDF:

$$f_{q(\bar{q})}^{\rm MC}(x,Q^2) = f_{q(\bar{q})}^{\overline{\rm MS}}(x,Q^2) + \int_x^1 \frac{dz}{z}\, f_{q(\bar{q})}^{\overline{\rm MS}}\left(\frac{x}{z},Q^2\right) \Delta C_{2q}(z) + \int_x^1 \frac{dz}{z}\, f_g^{\overline{\rm MS}}\left(\frac{x}{z},Q^2\right) \Delta C_{2g}(z)$$

where

$$\Delta C_{2q}(z) = C_{2q}^{\overline{\rm MS}}(z) - C_{2q}^{\rm MC}(z) = \frac{\alpha_s}{2\pi}\, C_F \left[\frac{1+z^2}{1-z} \ln\frac{(1-z)^2}{z} + 1 - z\right]_+$$

$$\Delta C_{2g}(z) = C_{2g}^{\overline{\rm MS}}(z) - C_{2g}^{\rm MC}(z) = \frac{\alpha_s}{2\pi}\, T_R \left\{\left[z^2 + (1-z)^2\right] \ln\frac{(1-z)^2}{z} + 2z(1-z)\right\}$$

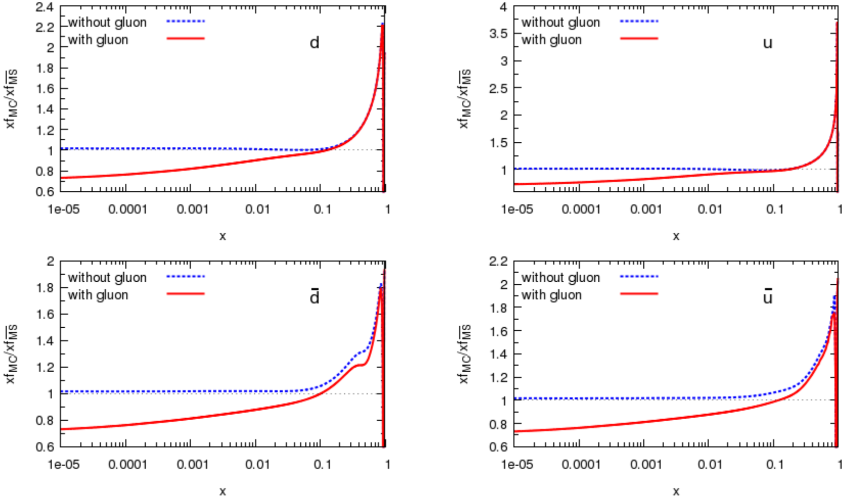
The formula is valid for any process up to $\mathcal{O}(\alpha_s^2)$.

The gluon PDF for DY

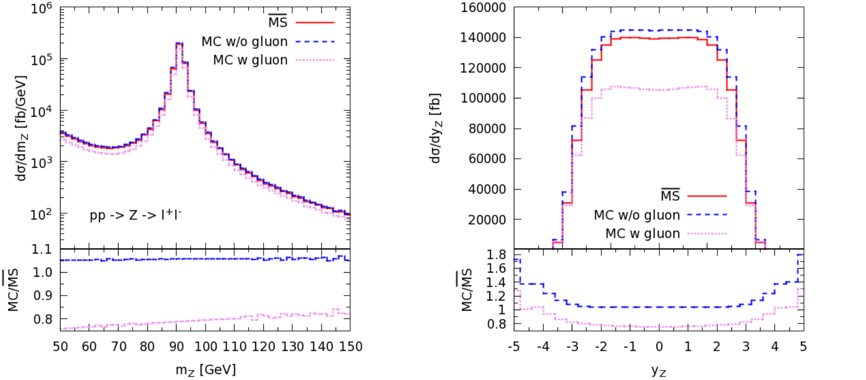
$$f_g^{\rm MC}(x,Q^2) = f_g^{\overline{\rm MS}}(x,Q^2)$$

MC PDFs

- Ratios with respect to standard $\overline{\mathrm{MS}}$ PDFs for light quarks.



$\overline{\text{MS}}$ vs MC at LO



- $+5\%$ effect at central rapidities in $q\bar{q}$ and -20% for both channels
- pronounced difference at large y coming from the $x \sim 1$ region

$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

$\overline{\text{MS}}$ scheme vs MC scheme at NLO

$$
\begin{aligned}
\sigma_{\text{tot}}^{\overline{\text{MS}}} &= f_q \otimes (1 + \alpha_s\, C_q^{\overline{\text{MS}}}) \otimes f_{\bar{q}} \\
\sigma_{\text{tot}}^{\text{MC}} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_q^{\text{MC}}) \otimes (f_{\bar{q}} + \alpha_s \Delta f_{\bar{q}}) \\
&= f_q f_{\bar{q}} + \alpha_s \left(\Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}} \right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3)
\end{aligned}
$$

At $\mathcal{O}(\alpha_s)$:

$$
C_q^{\overline{\text{MS}}} f_q f_{\bar{q}} = \Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}}
$$

Drell-Yan, $q\bar{q}$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$
(336.36 \pm 0.09)\,\text{pb} = \underbrace{25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb}}_{(336.35 \pm 0.09)\,\text{pb}}
$$

- Final result is scheme independent up to $\mathcal{O}(\alpha_s)$.
- A nontrivial test as $\Delta f_{q,\bar{q}}$ and C_2 come from totally different places. In particular, the former includes convolutions.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,\text{pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,\text{pb}$.

$\hookrightarrow$ Identical validation performed with both $q\bar{q}$ and qg channels.

Reweighting procedure

The "Sudakov" form factor for he CS shower

$$S(Q^2, \Lambda^2, x) = \int\limits_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int\limits_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

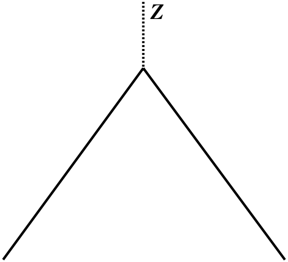
where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1+z^2}{1-z} \; \frac{D(q^2, x/z)/z}{D(q^2, x)}\,.$$

- z, q^2 - internal variables of the shower
- $D(q^2, x)$ - parton distribution functions

The kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution.

Upgrading to NLO: the hardest emission



$$\sigma^{\text{LO}} = \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus})$$

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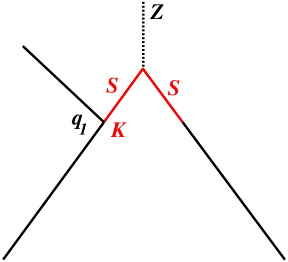

$$\sigma_{1+}^{\mathrm{PS}} = \sigma_B \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus)$$
$$\otimes \Big\{ S_\oplus(q_1^2, Q^2) K_\oplus(q_1^2, z_1) S_\ominus(q_1^2, Q^2) + S_\ominus(q_1^2, Q^2) K_\ominus(q_1^2, z_1) S_\oplus(q_1^2, Q^2) \Big\}$$

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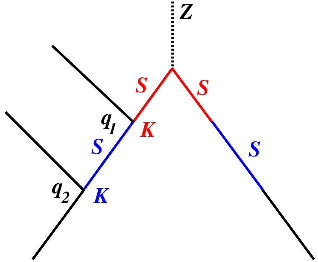

$$\sigma^{\rm PS}_{2+} = \sigma_B \otimes D_\oplus(Q^2,x_\oplus) \otimes D_\ominus(Q^2,x_\ominus)$$
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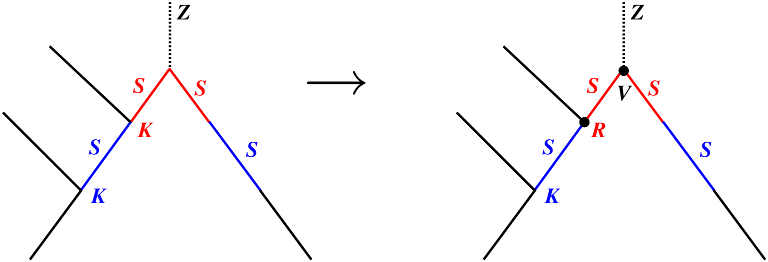

$$
\begin{aligned}
\sigma_{2+}^{\mathrm{NLO+PS}} = \; & \sigma_B \, (1+V) \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus) \\
& \otimes \Big\{ S_\oplus(q_1^2, Q^2) K_\oplus(q_1^2, z_1) S_\ominus(q_1^2, Q^2) \, R_\oplus(q_1^2, z_1)/K_\oplus(q_1^2, z_1) \\
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& \quad + S_\ominus(q_1^2, Q^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(q_1^2, Q^2) \, R_\ominus(q_1^2, z_1)/K_\ominus(q_1^2, z_1) \\
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\end{aligned}
$$

The MC weight for CS shower

↪ Averaged over angles of decay products.

Real part:

$$W_R^{q\bar{q}}(\alpha,\beta) = 1 - \frac{2\alpha\beta}{1+(1-\alpha-\beta)^2}$$

$$W_R^{qg}(\alpha,\beta) = 1 + \frac{\beta\,(\beta+2z)}{(1-z)^2+z^2}$$

Virtual + soft:

$$W_{V+S}^{q\bar{q}} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2}\right]$$

$$W_{V+S}^{qg} = 0$$

- **Real weight is a simple function of kinematic variables.** One can compute it on the fly, inside an MC, or outside, using information from event record.
- **Virtual+soft weight is a constant.** No need to generate strictly collinear contributions (like $d\Sigma^{c\pm}$ terms in MC@NLO).

NLO accuracy of KrkNLO results

$$d\sigma^{\text{KrkNLO}} = \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1+\Delta_{VS})e^{-S_F(s,q_s^2)}e^{-S_B(s,q_s^2)} \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B)$$

$$+ \int d^3\rho_1^F(sx_F x_B)(1+\Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)}D_{\text{MC}}^F(q_{1_F}^2, x_F)D_{\text{MC}}^B(q_{1_F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

$$+ \int d^3\rho_1^F(sx_F x_B)W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)}D_{\text{MC}}^F(q_{1_F}^2, x_F)D_{\text{MC}}^B(q_{1_F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

NLO accuracy of KrkNLO results

$$d\sigma^{\text{KrkNLO}} = \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1+\Delta_{VS})e^{-S_F(s,q_s^2)}e^{-S_B(s,q_s^2)} \times D_{\text{MC}}^F(q_s^2, x_F)D_{\text{MC}}^B(q_s^2, x_B)J_{\text{LO}}(x_F, x_B)$$

$$+ \int d^3\rho_1^F(sx_F x_B)(1+\Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)}D_{\text{MC}}^F(q_{1_F}^2, x_F)D_{\text{MC}}^B(q_{1_F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

$$+ \int d^3\rho_1^F(sx_F x_B)W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)}D_{\text{MC}}^F(q_{1_F}^2, x_F)D_{\text{MC}}^B(q_{1_F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

NLO accuracy of KrkNLO results

$$d\sigma^{\text{KrkNLO}} = \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1+\Delta_{VS})e^{-S_F(s,q_s^2)}e^{-S_B(s,q_s^2)} \times D^F_{\text{MC}}(q_s^2, x_F)D^B_{\text{MC}}(q_s^2, x_B)J_{\text{LO}}(x_F, x_B)$$

$$+ \int d^3\rho_1^F(sx_F x_B)(1+\Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)}D^F_{\text{MC}}(q_{1_F}^2, x_F)D^B_{\text{MC}}(q_{1_F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

$$+ \int d^3\rho_1^F(sx_F x_B)W^{[1]}_{q\bar{q}}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)}D^F_{\text{MC}}(q_{1_F}^2, x_F)D^B_{\text{MC}}(q_{1_F}^2, x_B)J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

- $D^{F,B}_{\text{MC}}(q_{1_F}^2, x_{F,B}) \longrightarrow D^{F,B}_{\text{MC}}(s, x_{F,B})$
- $e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)} \longrightarrow 1$

NLO accuracy of KrkNLO results

$$d\sigma^{\mathrm{KrkNLO}} = \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1+\Delta_{VS})e^{-S_F(s,q_s^2)}e^{-S_B(s,q_s^2)} \times D^F_{\mathrm{MC}}(q_s^2, x_F)D^B_{\mathrm{MC}}(q_s^2, x_B)J_{\mathrm{LO}}(x_F, x_B)$$

$$+ \int d^3\rho_1^F(sx_F x_B)(1+\Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \times e^{-S_F(s,q_{1F}^2)}e^{-S_B(s,q_{1F}^2)}D^F_{\mathrm{MC}}(q_{1F}^2, x_F)D^B_{\mathrm{MC}}(q_{1F}^2, x_B)J_{\mathrm{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)$$

$$+ \int d^3\rho_1^F(sx_F x_B)W^{[1]}_{q\bar{q}}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \times D^F_{\mathrm{MC}}(s, x_F)D^B_{\mathrm{MC}}(s, x_B)J_{\mathrm{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)$$

NLO accuracy of KrkNLO results

$$d\sigma^{\text{KrkNLO}} = \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1+\Delta_{VS})e^{-S_F(s,q_s^2)}e^{-S_B(s,q_s^2)} \times D_{\text{MC}}^F(q_s^2, x_F) D_{\text{MC}}^B(q_s^2, x_B) J_{\text{LO}}(x_F, x_B)$$

$$+ \int d^3\rho_1^F(sx_F x_B)(1+\Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times e^{-S_F(s,q_{1_F}^2)}e^{-S_B(s,q_{1_F}^2)} D_{\text{MC}}^F(q_{1_F}^2, x_F) D_{\text{MC}}^B(q_{1_F}^2, x_B) J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

$$+ \int d^3\rho_1^F(sx_F x_B) W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1_F}, \hat{\theta}) \times D_{\text{MC}}^F(s, x_F) D_{\text{MC}}^B(s, x_B) J_{\text{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)$$

NLO accuracy of KrkNLO results

$$d\sigma^{\mathrm{KrkNLO}} = \frac{d\sigma}{d\Omega}(sx_F x_B, \hat{\theta})(1+\Delta_{VS})e^{-S_F(s,q_s^2)}e^{-S_B(s,q_s^2)} \times D_{\mathrm{MC}}^F(q_s^2, x_F)D_{\mathrm{MC}}^B(q_s^2, x_B)J_{\mathrm{LO}}(x_F, x_B)$$

$$+\int d^3\rho_1^F(sx_F x_B)(1+\Delta_{VS})\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \times e^{-S_F(s,q_{1F}^2)}e^{-S_B(s,q_{1F}^2)}D_{\mathrm{MC}}^F(q_{1F}^2, x_F)D_{\mathrm{MC}}^B(q_{1F}^2, x_B)\Big[J_{\mathrm{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2) + J_{\mathrm{LO}}(x_F, x_B) - J_{\mathrm{LO}}(x_F, x_B)\Big]$$

$$+\int d^3\rho_1^F(sx_F x_B)W_{q\bar{q}}^{[1]}(k_1)\frac{d\sigma}{d\Omega}(sx_F x_B z_{1F}, \hat{\theta}) \times D_{\mathrm{MC}}^F(s, x_F)D_{\mathrm{MC}}^B(s, x_B)J_{\mathrm{NLO}}(x_F, x_B, z_{1F}, k_{1T}^2)$$

NLO accuracy of KrkNLO results

$$
\begin{aligned}
d\sigma^{\mathrm{KrkNLO}} = {} & \frac{d\sigma}{d\Omega}(s x_F x_B, \hat{\theta})(1+\Delta_{VS}) e^{-S_F(s,q_s^2)} e^{-S_B(s,q_s^2)} \\
& \times D^F_{\mathrm{MC}}(q_s^2, x_F) D^B_{\mathrm{MC}}(q_s^2, x_B) J_{\mathrm{LO}}(x_F, x_B) \\
& + \int d^3\rho_1^F(s x_F x_B)(1+\Delta_{VS}) \frac{d\sigma}{d\Omega}(s x_F x_B z_{1_F}, \hat{\theta}) \\
& \quad \times e^{-S_F(s,q_{1_F}^2)} e^{-S_B(s,q_{1_F}^2)} D^F_{\mathrm{MC}}(q_{1_F}^2, x_F) D^B_{\mathrm{MC}}(q_{1_F}^2, x_B) \Big[J_{\mathrm{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2) \\
& \qquad + J_{\mathrm{LO}}(x_F, x_B) - J_{\mathrm{LO}}(x_F, x_B) \Big] \\
& + \int d^3\rho_1^F(s x_F x_B) W^{[1]}_{q\bar{q}}(k_1) \frac{d\sigma}{d\Omega}(s x_F x_B z_{1_F}, \hat{\theta}) \\
& \quad \times D^F_{\mathrm{MC}}(s, x_F) D^B_{\mathrm{MC}}(s, x_B) J_{\mathrm{NLO}}(x_F, x_B, z_{1_F}, k_{1T}^2)
\end{aligned}
$$

- $+J_{\mathrm{LO}}$ term goes to the first part and moves PDFs to the scale s
- $-J_{\mathrm{LO}}$ term makes the second part finite $\Rightarrow$ drop Sudakovs and Δ_{VS}
- $s \longrightarrow \hat{s}$ (corrections $\mathcal{O}(\alpha_s^2)$)

NLO accuracy of KrkNLO results

We get

$$d\sigma^{\text{KrkNLO}} = (1+\Delta_{VS})\frac{d\sigma}{d\Omega}(s_1,\hat{\theta})\, J_{\text{LO}}\, D_F^{\text{MC}}(\hat{s},x_F)D_B^{\text{MC}}(\hat{s},x_B)$$
$$+ \int d^3\rho_1 \Big\{ \frac{d\sigma}{d\Omega}(zs_1,\hat{\theta})(1+W_{q\bar{q}}^{[1]})J_{\text{NLO}} - \frac{d\sigma}{d\Omega}(zs_1,\hat{\theta})\, J_{\text{LO}} \Big\}$$
$$\times D_F^{\text{MC}}(\hat{s},x_F)D_B^{\text{MC}}(\hat{s},x_B)$$

to be compared with

$$d\sigma^{\text{NLO}} = \int dz\, D_F^{\overline{\text{MS}}}(\hat{s},x_F)D_B^{\overline{\text{MS}}}(\hat{s},x_B)$$
$$\times \Big\{ (1+\Delta_{VS})\ \delta(1-z) + 2\Sigma_q(z) \Big\} \frac{d\sigma_0}{d\Omega}(zs_1,\theta)J(x_F,x_B,1,0)$$
$$+ \int \Big\{ d\sigma_{\text{NLO}} J_{\text{NLO}} - d\sigma_{\text{NLO-ct}}\ J_{\text{LO}} \Big\}\, D_F^{\overline{\text{MS}}}(\hat{s},x_F)D_B^{\overline{\text{MS}}}(\hat{s},x_B)$$

Results

NLO+PS results

KrkNLO

- Virtual: $\mu^2 = \mu_F^2 = \mu_R^2 = m_Z^2$
- Real: two choices
  - $\mu^2 = m_Z^2$
  - $\mu^2 = q^2$

  $\hookrightarrow$ differences formally beyond NLO, indicative of missing higher orders

Compared to:

- **MCFM**: pure NLO, $\mu^2 = m_Z^2$
- **MC@NLO**: from Sherpa, with the evolution variable q^2
- **POWHEG**: from Herwig++, with the evolution variable k_T^2

Matched results: total cross section

$q\bar{q}$ channel

	$\sigma_{\text{tot}}^{q\bar{q}}$ [pb]
MCFM	1273.4 ± 0.1
MC@NLO	1273.4 ± 0.1
POWHEG	1272.1 ± 0.7
KrkNLO $\alpha_s(q^2)$	1282.6 ± 0.2
KrkNLO $\alpha_s(M_Z^2)$	1285.3 ± 0.2

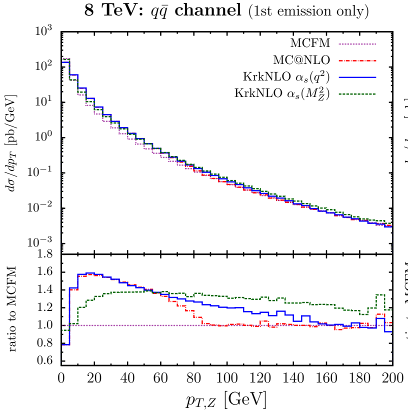
- sub-percent differences from beyond-NLO terms in the KrkNLO result (MC PDFs, mixed real-virtual)
- negligible difference between fixed and running coupling

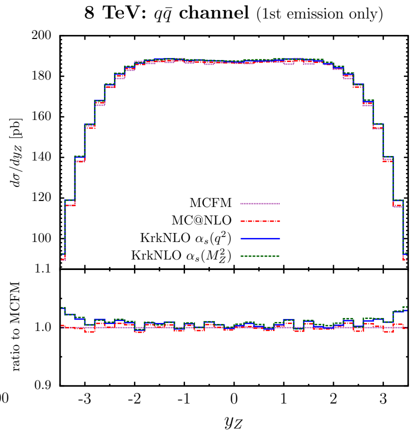
$q\bar{q} + qg$ channels

	$\sigma_{\text{tot}}^{q\bar{q}+qg}$ [pb]
MCFM	1086.5 ± 0.1
MC@NLO	1086.5 ± 0.1
POWHEG	1084.2 ± 0.6
KrkNLO $\alpha_s(q^2)$	1045.4 ± 0.1
KrkNLO $\alpha_s(M_Z^2)$	1039.0 ± 0.1

- beyond-NLO terms reach up to 4% in the KrkNLO result
  $\hookrightarrow$ resulting from large gluon luminosity leading to $f^{\text{MC}}/f^{\overline{\text{MS}}} < 1$
- small differences between fixed and running coupling choices

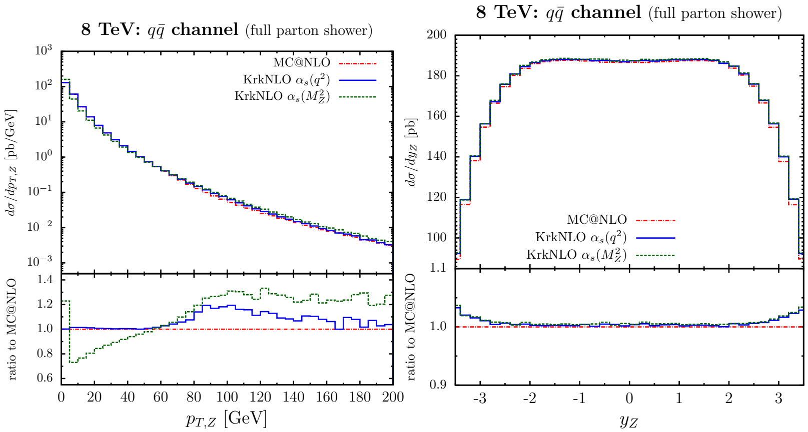
Matched results: $q\bar{q}$, 1st emission





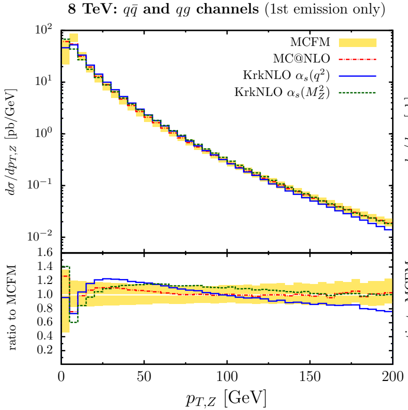
- Reproduction of y_Z distribution at NLO.
- Agreement of KrkNLO $\alpha_s(q^2)$ with MC@NLO at low $p_{T,Z}$: PS domination
- KrkNLO results above MC@NLO and MCFM at higher $p_{T,Z}$: $\mathcal{O}(\alpha_s^2)$ terms

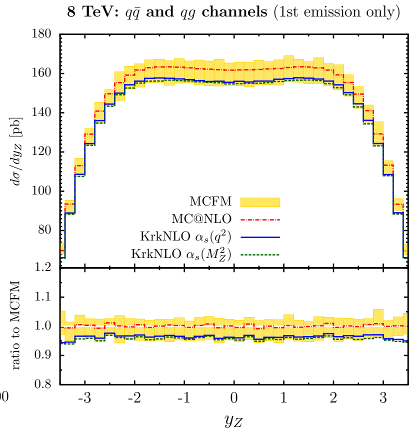
Matched results: $q\bar{q}$, full PS



- Low $p_{T,Z}$ part of the spectrum changes but KrkNLO $\alpha_s(q^2)$ with MC@NLO agree there because of shower domination
- KrkNLO results above pure NLO at high $p_{T,Z}$: admixture of NNLO terms
- Diffs between two KrkNLO result at high $p_{T,Z}$: running coupling effects

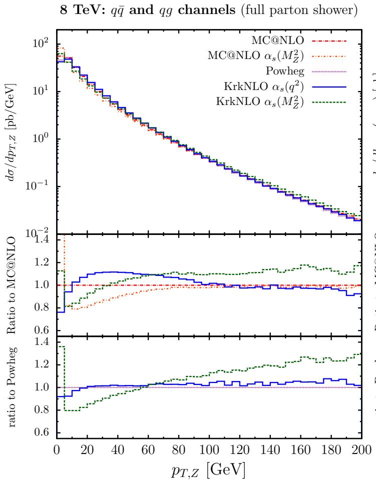
Matched results: botch channels, 1st emission

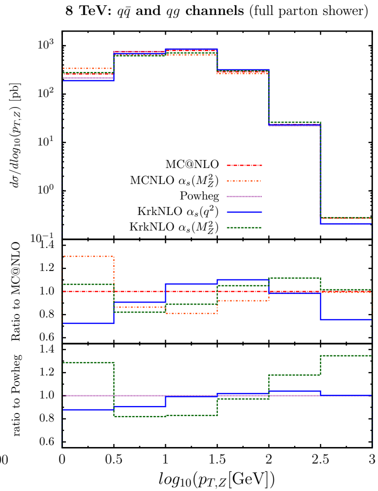




- Moderate differences between KrkNLO $\alpha_s(q^2)$ and MC@NLO in the region below M_Z and between KrkNLO $\alpha_s(M_Z^2)$ and MC@NLO in the region above M_Z

Matched results: both channels, full PS





- KrkNLO $\alpha_s(q^2)$ stays overall very close to MC@NLO
- KrkNLO $\alpha_s(q^2)$ almost coincides with POWHEG $p_{T,Z}$ distributions

Conclusions

- A new method of NLO+PS matching was presented:
  - Events of LO parton shower are corrected by simple reweighting.
  - Pathological collinear NLO terms are dealt with by means of transfering them into PDFs:
    The change of factorization scheme from $\overline{\text{MS}}$ into MC.
  - Virtual correction is just a constant, independ of kinematics.
- The implementation on top of Catani-Seymour shower in Sherpa event generator was presented.
- Careful validated against fixed order NLO for Drell-Yan process in the analytical and numerical form was done.
- Examples of comparisons to MC@NLO and POWHEG were shown.