The Monte-Carlo tools for the Drell-Yan production of W and Z bosons for the precision measurements era

Collinear Factorization and parton shower Monte Carlo

S. JADACH

A. Kusina, M. Skrzypek and M. Sławińska

IFJ-PAN, Kraków, Poland

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More on http://jadach.web.cern.ch/

Introduction

Parton shower MC (psMC) is a workhorse in the data analysis in hadron colliders

THE CHALLENGE:

Can we put more of pQCD into parton shower MCs? And even PDFs?

QCD and pQCD

Last $\sim$40 years

- **1954–75:** establishing QCD as renormalizable QFT with asymptotic freedom
- **1976–83:** "Gold-rush epoch". Principles of practical pQCD calculations; CFTs, PDFs, NLO at e^+e^-, DIS, DY (but experiment 20 years behind theory!).
- **1984-2002:** Experimental tests at at PEP/PETRA, Tevatron, LEP and consolidation of relevant th. tools.
- **2002-Now:** Important/crucial role of QCD – helping data analysis at hadron colliders. Extending/refining practical toolbox of pQCD.

pQCD and Monte Carlo

- **1974-84:** parton shower MC (psMC) simulating hadronization only.
- **1984/85:** psMC simulates LO/LL evolution of the parton distributions (DGLAP).
- **1976-Now:** MC used to integrate tree-level multiparton phase space, recently including NLO corrections to hard process.
- **2002-Now:** Methods of combining LO psMC with LO+NLO hard process (MC@NLO...)
- **1985-Now:** psMC stays at LO/LL level; Why?????!!!!

Why psMC stays at LO since 1985?

Possible answers before ∼2004:

- Not important/urgent (poor exp. data).
- Unfeasible, or at least extremely difficult.

Both above answers are now invalid:

- LHC data analysis will require improved QCD, NLO psMC in particular.
- Krakow group shows that NLO psMC is feasible:-) Albeit difficult:-(

Krakow (IFJPAN) R&D activity in pQCD, 2004-now

- **2004-2007:** Constructing new variants of LO parton shower MC: Constrained MC algorithm, CCFM, inclusive NLO kernels, first studied on YFS-style inclusion NLO corrs. to hard process.
- **2007-2010:** Constructing NLO parton shower MC: re-calculating NLO kernels (non-singlet), working out 2 scenarios for including NLO corrections in exclusive form, studies on soft non-abelian limit of exclusive LO+NLO kernels, studies of factorization scheme dependence
- **2010-2011:** New YFS-like schemes of including NLO in hard process ME in W/Z production at LHC and DIS at HERA (alternative to MC@NLO) and in the ladder/uPDF.

S-J, K.Golec-Biernat, A.Kusina, M.Skrzypek, W.Placzek, M.Slawinska, P.Stephens, P.Stoklosa, Z.Wąs.

Main lessons from Krakow R&D effort

- **Backward evolution MC algorithm for initial state parton shower is not the only solution (important technical point)**
- **QCD Collinear Factorization Theorems (CFTs) to be reorganized before pushing psMC beyond LO:**
  (a) Violation of 4-mom. to be repaired
  (b) Explicit time-ordered exponent in the phase space,
- **Soft gluon limit beyond LO dictates angular ord.**
- **LO psMC has to be reorganized before NLO included.**
- **Inevitable departure from standard $\overline{MS}$, CFP scheme:**
  extra term $\alpha_S P(z)\ln(1-z)$ **in coeff. function** C_2 **and**
  $\alpha_S \int_{z=z_1 z_2} P(z_1)P(z_2)\ln(z_1(1-z_2)/(1-z_1))$ **in NLO kernels.**
- **New YFS-like scheme for NLO cors. to hard process – an attractive alternative to MC@NLO and POWHEG?**

Even more...

psMC can absorb many objects and techniques of pQCD presently in non-MC (or analytical) world:

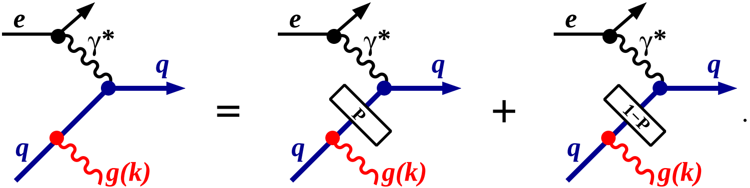
- **Universal PDFs, their definition, $\ln(Q)$ evolution, and even extraction from the data,**
- **kT factorization/resummation,**
- **low-x modelling (CCFM, BFKL),**
- **finite quark mass thresholds,**
- **soft gluon resummations of $\frac{\ln^n(1-x)}{(1-x)}$ etc., both in the hard process and the ladder.**

Main OBSTACLE on the way to new psMC: Collinear Factorization Theorems (CFTs)

- **Classic CFTs** were formulated by:
  △ 1979, EGMPR [Ellis, Georgi, Machacek, Politzer, Ross],
  △ 1980, CFP [Curci, Furmanski, Petronzio],
  △ 1980-85, CSS [Collins, Sterman , Soper, Bodwin,...]
- **Why not suited** for the parton shower MC?
  ⋆ **Non-conservation of the 4-momenta**
  ⋆ **Over-subtractions**
- **How to modify CFT?** for use in psMC:
  • Redefine projection operators for extracting singular (Coll.) parts, such that 4-mom. is conserved.
  • Introduce time-ordered exponential earlier, while isolating/subtracting Coll/IR singular parts.

Non-conservation of 4-momentum in CFTs
The culprit is LO-extracting “casting” operator $\mathbb{P}$ of CFTs

Exact = LO + NLO



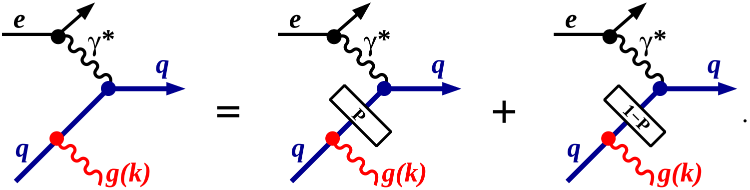
$$\int_{m_p}^{TruePhSp.} \frac{dk_T}{k_T}(\ldots) = \int dk_T(\ldots)\delta(k_T)\int_{m_p}^{\mu_F} \frac{dk'_T}{k'_T} \quad + \ldots$$

$\int dk_T$ above and below $\mathbb{P}$ decouples.

Non-conservation of 4-momentum in CFTs

The culprit is LO-extracting "casting" operator $\mathbb{P}$ of CFTs

Exact = LO + NLO

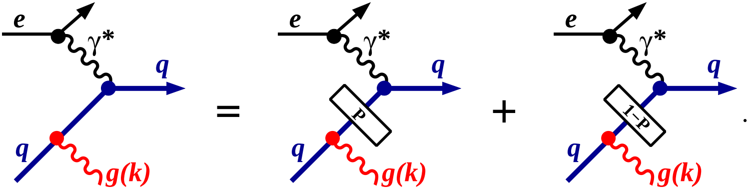


$$\int_{m_p}^{TruePhSp.} \frac{dk_T}{k_T}(\dots) = \int dk_T(\dots)\delta(k_T)\ln\frac{\mu_F}{m_p} + \dots$$

Explicit collinear big logarithm.

Non-conservation of 4-momentum in CFTs
The culprit is LO-extracting “casting” operator $\mathbb{P}$ of CFTs

Exact = LO + NLO

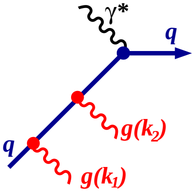


$$\int_0^{PhSp.} \frac{dk_T}{k_T} \left(\frac{k_T}{\mu_F}\right)^{\varepsilon} = \frac{1}{\varepsilon} + (1 - PP) \int_0^{PhSp.} \frac{dk_T}{k_T} \left(\frac{k_T}{\mu_F}\right)^{\varepsilon}$$

Dimensional regularization obscures the picture; $\delta(k_T)$ implicit.

The real disaster strikes in case of 2 gluons!

True result from Feynman diagram is:

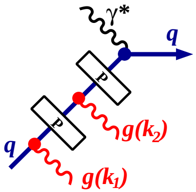


$$\longrightarrow \quad \frac{1}{2}\ln^2\frac{\mu_F}{m_p}$$

Over-subtractions, over-cancellations in CFTs
The real disaster strikes for 2 gluons!

Apparently wrong LO! **In addition to k_T non-conservation:**

Double use of casting operator yields wrong LO:



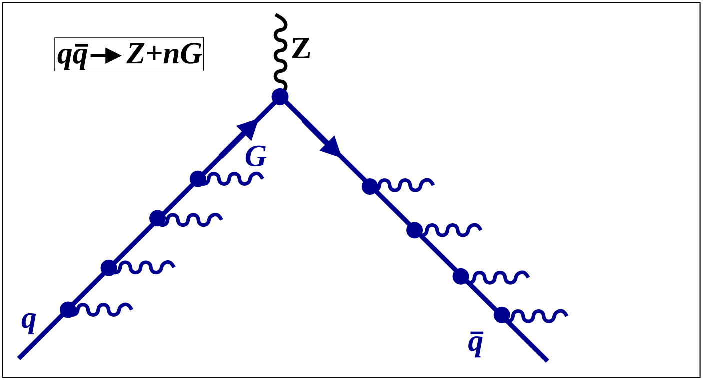
$$\longrightarrow \quad 1 \ln^2 \frac{\mu_F}{m_p}$$

WAIT! CFTs self-correct for the above LO over-estimate!
– But in a way which inhibits MC implementation of CFTs.

Lets go to full size CFT...

EGMPR factorization theorem/scheme

R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer and G. G. Ross,
Nucl. Phys. B 152, 285 (1979). See also J. Collins Phys.Rev. D58, 1998



Gluostrahlung diagram

EGMPR factorization theorem/scheme

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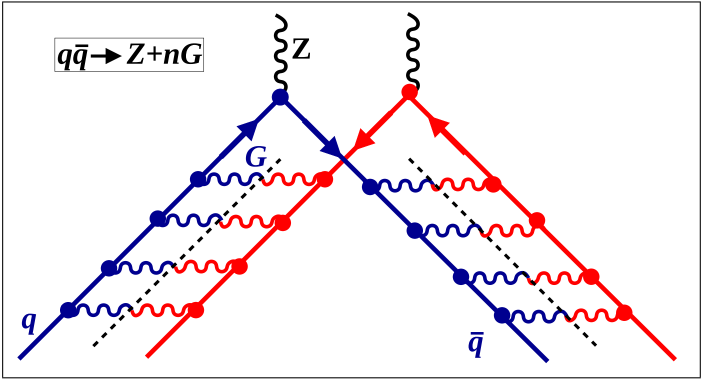
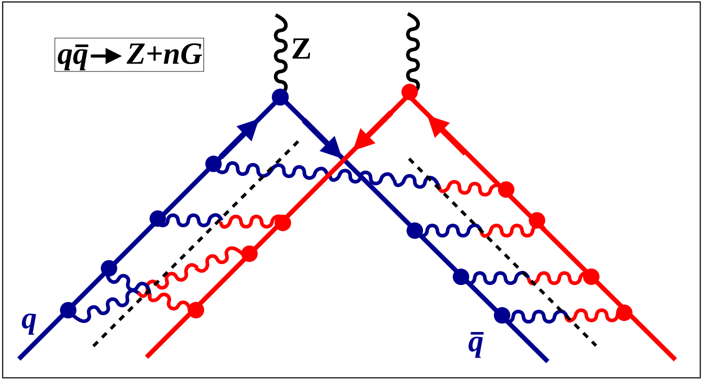


Gluostrahlung diagram **squared**

EGMPR factorization theorem/scheme

R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer and G. G. Ross, Nucl. Phys. B 152, 285 (1979). See also J. Collins Phys.Rev. D58, 1998



Gluostrahlung diagram squared +interferences

EGMPR factorization theorem/scheme

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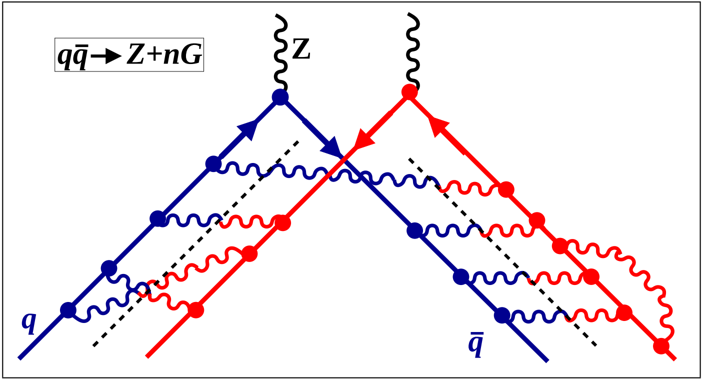
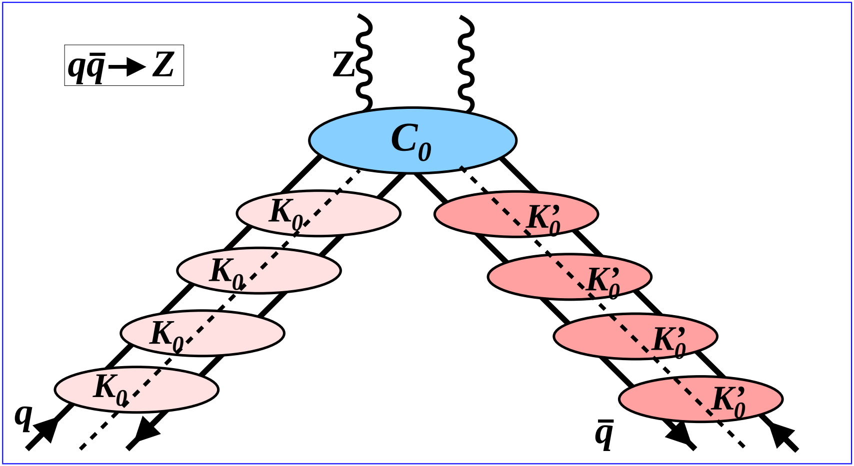


Gluostrahlung diagram squared +interferences +virtuals

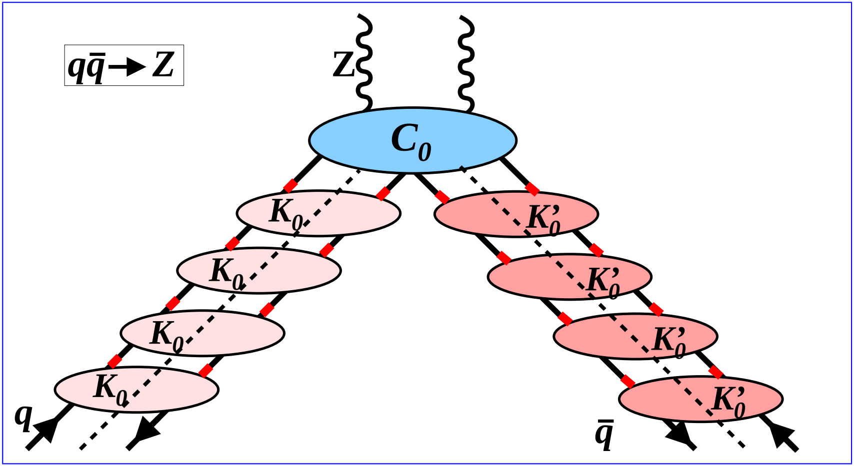
EGMPR raw factorization theorem
Infinite order analysis if coll. singularities in axial gauge



- C_0 is finite, K_0, K'_0 are 2PI kernel. Phase space intact!
- Vertical rungs | are source of ALL Coll. singularities.

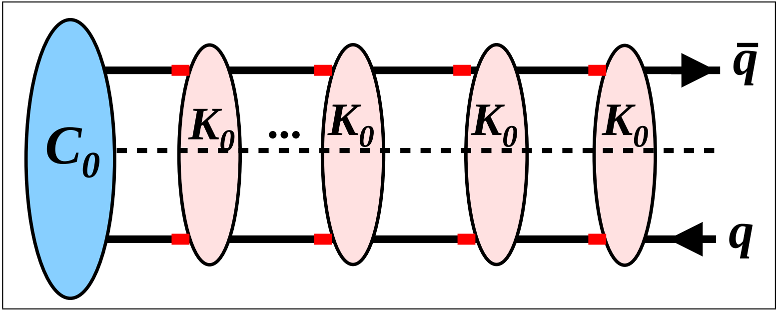
EGMPR raw factorization theorem

Infinite order analysis if coll. singularities in axial gauge



- C_0 is finite, K_0, K'_0 are 2PI kernel. Phase space intact!
- Vertical rungs **|** are source of ALL Coll. singularities.

EGMPR factorization theorem/scheme
Single ladder

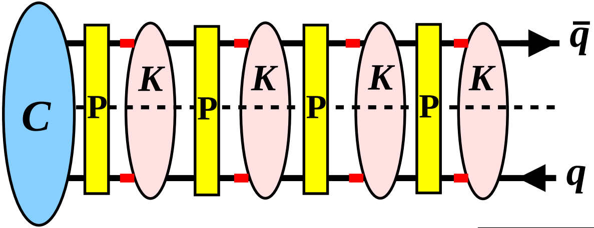


$$F = C_0 \cdot \frac{1}{1-K_0} = C_0 \cdot (1 + K_0 + K_0 \otimes K_0 + K_0 \otimes K_0 \otimes K_0 + \ldots),$$

Curci-Furmanski-Petronzio (CFP) collinear factorization scheme (1979)

CFP customized EGMPR to $\overline{MS}$ and exploited it to NLO in practice. Using **casting operator** $\mathbb{P} = P_{spin}\, PP$ they get:

$$F = C \cdot \frac{1}{1-K} = C_0 \cdot (1 + K + K \otimes K + K \otimes K \otimes K + ...)$$



where new **finite hard process part** is: $C = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}$

The ladder encapsulates all collinear singularities.

Reorganized kernel is: $K = \mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}$

Over-subtraction problem in translating CFP/EGMPR into Monte Carlo

From renormalization group eqs. (RGE) or explicit LO calc. we know:

$$\Gamma = e^{+\frac{1}{\varepsilon}} = 1 + \frac{1}{\varepsilon} + \frac{1}{2!}\frac{1}{\varepsilon^2} + ...$$

Examining up to LO real emissions in CFP scheme we see enormous over-subtractions/cancellations:

$$\Gamma \simeq \frac{1}{1 - \left(1 - e^{-\frac{1}{\varepsilon}}\right)} = 1 + \left(1 - e^{-\frac{1}{\varepsilon}}\right) + \left(1 - e^{-\frac{1}{\varepsilon}}\right)^2 + ...$$

NO WAY to build Monte Carlo on that!
Need exponent directly from the Feynman diagrams!!!

Translating ε-poles $\frac{1}{\varepsilon} = \int_0^{\mu_F} \frac{dk^T}{k^T} \left(\frac{k^T}{\mu_F}\right)^{\varepsilon}$ into ordinary big logs $\ln \frac{\mu_F}{m_p}$ of EGMPR provides the same picture.

Correcting for over-subtractions
New CFT suited for the psMC?

Over-subtraction eliminated thanks to explicit time ordered exponential= $\exp_{TO}$ in the evolution variable = log of the factorization scale (Soper+Nagy at LO):

$$F = \frac{1}{1-K_0} = C_0 \overleftarrow{\mathbb{B}}_\mu \left[\frac{1}{1-K_0}\right] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'_\mu \left\{ {}^s K_0 \cdot \overleftarrow{\mathbb{B}}_s \left[\frac{1}{1-K_0}\right]\right\}\right)$$

$$\overleftarrow{\mathbb{B}}_\mu \left[\frac{1}{1-K_0}\right] \equiv 1 + \overleftarrow{\mathbb{B}}_\mu[K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] + \dots$$

Operator $\overleftarrow{\mathbb{B}}$ is defined recursively (similarly as β-functions in Yennie-Frautschi-Suura 1961 subtraction scheme):

$$\overleftarrow{\mathbb{B}}_\mu[K_0] = K_0 - \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] = K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \mathbb{P}'_\mu\{{}^{s_2}K_0 \cdot \overleftarrow{\mathbb{B}}_{s_2}[K_0]\} - \overleftarrow{\mathbb{B}}_\mu[K_0] \cdot \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] = K_0 \cdot K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_3}K_0\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \dots$$

Modified $\mathbb{P}'_\mu$ new projection operator is the key point!
$\mathbb{P} \rightarrow \mathbb{P}'_\mu$ conserves four-momentum, contrary to EGMPR.

Specs of $\overleftarrow{\mathbb{P}}'_\mu$ modified projection operator

- $\overleftarrow{\mathbb{P}}'_\mu$ does spin projection as $\mathbb{P}$ of CFP,
- $\overleftarrow{\mathbb{P}}'_\mu(A)$ extracts singular part from integrand A, (not from the integral $\int A$ like CFP!)
- where A is *at most single-log* coll. divergent!
- $\overleftarrow{\mathbb{P}}'_\mu$ acts on integrand, leaves out Lorentz inv. phase space, sets on-shell all (cut) real momenta towards the hard process
- $\overleftarrow{\mathbb{P}}'_\mu$ sets upper limit μ on the phase space for all real (cut) partons towards the hadron using kinematic variable $s(k_1, .., k_n) < \mu$,
- examples of $s(k_1, .., k_n)$: Virtuality, maximum rapidity $s = \max(k_i^T/\alpha_i)$, or $s = \max(k_i^T)$,
- Nesting like $\overleftarrow{\mathbb{P}}'_\mu[K_0 \cdot (1 - \overleftarrow{\mathbb{P}}'(K_0))]$ is allowed.

Hierarchy of fact. scales in T.O. expon.

Example for up to 3 terms:

$$\exp_{TO}\left(\mathbb{P}'_\mu\{A\}\right)(\mu) = 1 + \mathbb{P}'_\mu\{A\} + \mathbb{P}'_\mu\{^{s_2}A\}\cdot\mathbb{P}'_{s_2}\{^{s_1}A\} + \mathbb{P}'_\mu\{^{s_3}A\}\cdot\mathbb{P}'_{s_3}\{^{s_2}A\}\cdot\mathbb{P}'_{s_2}\{^{s_1}A\} + \dots$$

For $A = \int dLips(k_1, k_2, ..., k_n)\, f(k_1, ..., k_n)$,
with k_i being on-shell emitted partons,
notation $\{^{s_3}A\}$ defines $s_3 = a(a_1, ..., a_n) = \max(a_1, ..., a_n)$.
The entire integrand in

$$\mathbb{P}'_\mu\{^{s_3}A\}\cdot\mathbb{P}'_{s_3}\{^{s_2}A\}\cdot\mathbb{P}'_{s_2}\{^{s_1}A\}$$

is multiplied by

$$\theta_{\mu>s_3>s_2>s_1}$$

insted of EGMPR/CFP common limit:

$$\theta_{\mu>s_3}\,\theta_{\mu>s_2}\,\theta_{\mu>s_1}$$

Standard inclusive PDFs, evolution eqs. and kernels recovered after Ph.Sp. integration

Exclusive PDF (ePDF) is the integrand in:

$$D(\mu) = \exp_{TO}\left(\overleftarrow{\mathbb{P}}'_\mu \left\{ {}^s K_0 \cdot \overleftarrow{\mathbb{B}}_s \left[\frac{1}{1-K_0} \right] \right\} \right) = \exp_{TO}(K).$$

LO and NLO truncations of the **exclusive evolution kernel** K_μ are:

$$K^{LO}_\mu = \overleftarrow{\mathbb{P}}'_\mu \{ {}^s K_0 \}, \quad \text{taken at } \mathcal{O}(\alpha^1),$$

$$K^{NLO}_\mu = \overleftarrow{\mathbb{P}}'_\mu \left\{ {}^s K_0 + K_0 \cdot (1 - \overleftarrow{\mathbb{P}}'_s) \cdot K_0 \right\}, \text{ truncated at } \mathcal{O}(\alpha^2).$$

Standard inclusive PDF $D(\mu, x)$, from ph.space integration of ePDF with fixed x, obeys by construction DGLAP evolution equation:

$$\partial_\mu D(\mu, x) = \mathcal{P} \otimes D(\mu)(x)$$

with the DGLAP **standard inclusive kernel**:

$$\mathcal{P}(x) = \int d\mathrm{Lips}\, \delta\left(x = \frac{\sum k_i^+}{E_0} \right) \delta\left(1 - \frac{s}{\mu} \right) \overleftarrow{\mathbb{P}}'_\mu \left\{ {}^s K_0 \cdot \overleftarrow{\mathbb{R}}_s [K_0] \right\}.$$

Bottom line:

Modified factorization scheme cures both problems of EGMPR/CFP:

(i) over-subtraction/cancellations and

(ii) 4-momentum non-conservation.

Ready to go for psMC with complete NLO, both in the hard part and ladder parts.

Important ingredient to be added: kinematic mapping in the modified $\overleftarrow{\mathbb{P}}'_{\mu}$, see next slides on psMC for DY.

NLO psMC for:

W/Z production (Drell-Yan)

and for ep DIS

NLO psMC for W/Z production at LHC/Tevatron

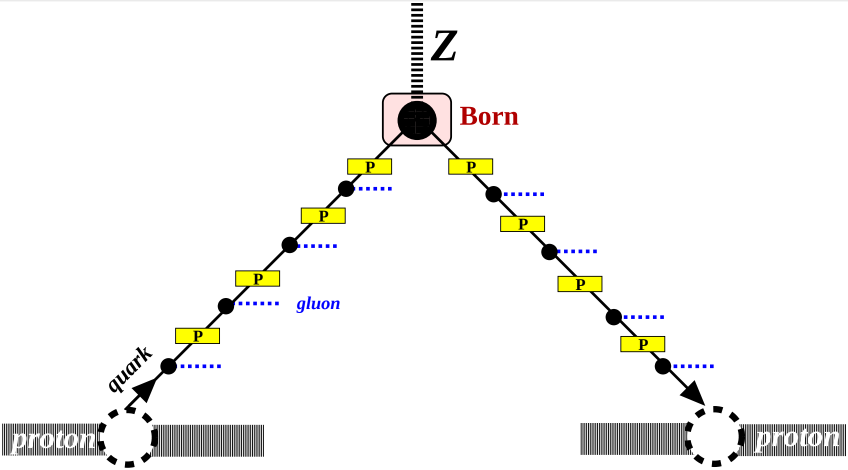
In the following several simplifications are adopted temporarily:

- Non-singlet kernels only,
- Only $q\bar{q} \rightarrow W/Z$, omitted $qg \rightarrow W/Z$
- non-running α_S
- Initial PDFs at low $\mu = Q$ assumed, but not explicitly shown in the formulae

The notorious gluonstrahlung is our primary target!

LO psMC is (re-)constructed from the scratch, in a way compatible with our new factorization scheme.

LO psMC is (re-)constructed from the scratch



$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\left\{\sigma\left[C_0^{(0)}(\mathbb{P}'K_{0F}^{(1)})^{n_1}(\mathbb{P}''K_{0B}^{(1)})^{n_2}\right]\right\}_{T.O.}$$

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\, d\hat{x}_B\, d\Xi$$
$$\times\, e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}$$
$$\times\, e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}$$
$$\times\,\delta\left(\Xi-\frac{\eta_{0F}+\eta_{0B}}{2}-\ln\frac{\hat{x}_F}{\hat{x}_B}\right) d\tau_2\Big(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2\Big)\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),$$

S_F and S_B = Sudakov formfactors, $\bar{P}(z)=\frac{1}{2}(1+z^2)$,
Ξ = Rapidity of Z, division plane between F and B hemispheres.
θ = angle of decay products (leptons) in Z rest frame.
$\hat{s}=s\hat{x}_F\hat{x}_B$ = effective mass of Z boson.

LO psMC for W/Z production: Details

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\; d\hat{x}_B\; d\Xi$$

$$\times\; e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\;\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j}$$

$$\times\; e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\;\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j}$$

$$\times\;\delta\left(\Xi-\frac{\eta_{0F}+\eta_{0B}}{2}-\ln\frac{\hat{x}_F}{\hat{x}_B}\right)\;d\tau_2\Big(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2\Big)\;\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),$$

Eikonal phase space for real gluon:
$d^3\mathcal{E}(k)=\frac{d^3k}{2k^0}\,\frac{1}{\mathbf{k}^2}=\pi\frac{d\phi}{2\pi}\frac{d\alpha}{\alpha}d\eta=\pi\frac{d\phi}{2\pi}\frac{d\beta}{\beta}d\eta,$
Lightcone variables: $\alpha=\frac{k^+}{2E},\;\beta=\frac{k^-}{2E}$; rapidity: $\eta=\frac{1}{2}\ln\frac{k^+}{k^-}$,
$d\tau_2(Q;q_1,q_2)$ = two-body phase space element.

LO psMC for W/Z production: Details

$$
\begin{aligned}
\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int d\hat{x}_F\; d\hat{x}_B\; d\Xi \\
\times\, e^{-S_F}\int_{\Xi>\eta_{n_1}}\left(\prod_{i=1}^{n_1} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i>\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Fi})\right)\delta_{\hat{x}_F=1-\sum_j\hat{\alpha}_j} \\
\times\, e^{-S_B}\int_{\Xi<\eta_{n_2}}\left(\prod_{i=1}^{n_2} d^3\mathcal{E}(\bar{k}_i)\theta_{\eta_i<\eta_{i-1}}\frac{2C_F\alpha_s}{\pi^2}\bar{P}(z_{Bi})\right)\delta_{\hat{x}_B=1-\sum_j\hat{\beta}_j} \\
\times\, \delta\left(\Xi-\frac{\eta_{0F}+\eta_{0B}}{2}-\ln\frac{\hat{x}_F}{\hat{x}_B}\right) d\tau_2\Big(P-\sum_{j=1}^{n_1+n_2}k_j;q_1,q_2\Big)\,\frac{d\sigma_B}{d\Omega}(s\hat{x}_F\hat{x}_B,\hat{\theta}),
\end{aligned}
$$

Variables in LO evolution kernels:

$z_{Fi}=\frac{\hat{x}_{Fi}}{\hat{x}_{F(i-1)}},\;\hat{x}_{Fi}=1-\sum_{j=1}^{i}\hat{\alpha}_i=\prod_{j=1}^{i}z_{Fj},$

$z_{Bi}=\frac{\hat{x}_{Bi}}{\hat{x}_{B(i-1)}}\;\hat{x}_{Bi}=1-\sum_{j=1}^{i}\hat{\beta}_i=\prod_{j=1}^{i}z_{Bj},$

For “mapped” lightcone variables $\hat{\alpha}_i$ and $\hat{\beta}_i$ see next slide.

Order ALL gluons according to rapidity distance from Ξ, the position of Z: Define permutation $\pi = \{\pi_1, \pi_2, ..., \pi_{n_1+n_2}\}$ such that $|\eta_{\pi_i} - \Xi| > |\eta_{\pi_{i-1}} - \Xi|, \quad i = 1, ..., n_1 + n_2$

Define in a *recursive* way dilatation transformation:

$$k_{\pi_i} = \lambda_i \bar{k}_{\pi_i}, \quad \lambda_i = \frac{s(\bar{x}_{i-1} - \bar{x}_i)}{2(P - \sum_{j=1}^{i-1} k_{\pi_j}) \cdot \bar{k}_{\pi_i}}, \quad i = 1, 2, ..., n_1 + n_2.$$

Rescaling factor λ_i is chosen such that

$$\bar{s}_i = s\bar{x}_i = s\prod_{j=1}^{i} \hat{z}_{(F,B)\pi_j} = (P - \sum_{j=1}^{i} k_{\pi_j})^2 = (P - \sum_{j=1}^{i} \lambda_j \bar{k}_{\pi_j})^2.$$

In F hemisphere $\hat{\alpha}_i = \lambda_i \alpha_i$ and in B hemisphere $\hat{\beta}_i = \lambda_i \beta_i$.

(An improvement over 1st such scenario publ. in 2007 in APP, Stephes at.al.)

Overview of the LO Monte Carlo algorithm:

- Variables $\hat{z}_F$ and $\hat{z}_B$ are generated by FOAM and Ξ is determined immediately.
- Four momenta $\bar{k}_i^\mu$ are generated separately in F and B parts of the phase space using CMC module, with the corresponding constraints $\sum_{j\in F} \hat{\alpha}_j = 1 - \hat{z}_F$ and $\sum_{j\in B} \hat{\beta}_j = 1 - \hat{z}_B$.
- Double ordering permutation π is established.
- Using P and $\bar{k}_{\pi_1}$ rescaling parameter λ_1 is calculated, $k_{\pi_1} = \lambda_1 \bar{k}_{\pi_1}$ is set. At this stage $(P - k_{\pi_1})^2 = sx_1$, where $x_1 = z_{\pi_1} = 1 - \hat{\alpha}_{\pi_1}$ or $x_1 = z_{\pi_1} = 1 - \hat{\beta}_{\pi_1}$, depending whether k_{π_1} was in F or B part of LIPS.
- Using $P - k_{\pi_1}$ and $\bar{k}_{\pi_2}$ parameter λ_2 is found and $k_{\pi_2} = \lambda_2 \bar{k}_{\pi_2}$ is set. At this stage we enforce $(P - k_{\pi_1} - k_{\pi_2})^2 = sx_2 = sz_{\pi_1} z_{\pi_2}$. This recursive procedure continues until the last gluon.
- In the rest frame of $\hat{P} = P - \sum_j k_{\pi_j}$ 4-momenta of q_1^μ and q_2^μ are generated according to Born angular distribution.

Kinematics of the two hemispheres is interrelated very gently, starting from very collinear gluons and finishing with the least collinear ones.

Exact analytical integration

The above LO MC covers multigluon phase space without any gaps or overlaps.

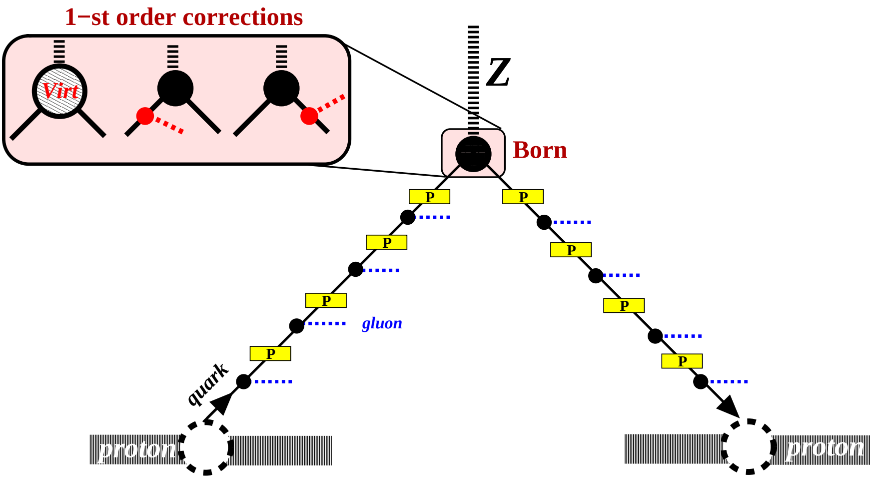
Moreover, EXACT analytical integration of the LO MC distributions over the multigluon phase space is possible:

$$\sigma(C_0^{(0)}\Gamma_F^{(1)}\Gamma_B^{(1)}) = \int_0^1 d\hat{x}_F\; d\hat{x}_B\; D_F(\Xi, \hat{x}_F)\; D_B(\Xi, \hat{x}_B)\; \sigma_B(s\hat{x}_F\hat{x}_B).$$

with two PDFs obeying DGLAP nonsinglet LO evolution equation:

$$\frac{\partial}{\partial \Xi} D_F(\Xi, x) = [\mathcal{P} \otimes D_F(\Xi)](x).$$

psMC with LO ladder and NLO hard process



NLO Monte Carlo weight
This is Yennie-Frautschi-Suura (YFS) style!

Once LO MC is re-designed, introduction of the complete NLO to hard process part is done with help of simple positive MC weight:

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Fj})}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_F,\hat{p}_B;a_j,z_{Bj})}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

where the IR/Col.-finite real emission part is

$$\tilde{\beta}_1(\hat{p}_F,\hat{p}_B;q_1,q_2,k) = \left[\frac{(1-\alpha)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{F1}) + \frac{(1-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\theta_{B2})\right]$$
$$-\theta_{\alpha>\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}) - \theta_{\alpha<\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s},\hat{\theta}),$$

and the kinematics independent virtual+soft correction is

$$\Delta_{V+S} = \frac{C_F\alpha_s}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F\alpha_s}{\pi}\,\frac{1}{2}$$

Next slide more on Δ_{V+S}.

More on Δ_{V+S} virtual+soft correction

$$\Delta_{V+S} = D_{DY}^{\overline{MS}}(z) - 2C_{ct}^{psMC}(z)$$

where we use $\overline{MS}$ results, eq. (89) in Altarelli+Ellis+Martinelli (1979):

$$D_{DY}^{\overline{MS}}(z), = \delta(1-z) \; + \delta(1-z)\frac{C_F\alpha_s}{\pi} \left(\frac{1}{3}\pi^2 - 4\right) +$$
$$+ 2\frac{C_F\alpha_s}{\pi} \left(\frac{\hat{s}}{\mu^2}\right)^{\varepsilon} \left(\frac{\bar{P}(z)}{1-z}\right)_+ \left(\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + [2\ln(1-z) - \ln z]\right)$$

and collinear counterterm of psMC (one gluon in psMC in $d = 4 + 2\varepsilon$):

$$C_{ct}^{psMC}(z) = \frac{C_F\alpha_s}{\pi} \int_{\beta<\alpha} \frac{d\alpha d\beta}{\alpha\beta} \int d\Omega_{1+2\varepsilon} \left(\frac{s\alpha\beta}{\mu_F^2}\right)^{\varepsilon} \bar{P}(1-\alpha,\varepsilon)\delta_{1-z=\alpha} =$$
$$= \frac{C_F\alpha_s}{\pi} \left(\frac{\bar{P}'(z,\varepsilon)}{1-z}\right)_+ \left(\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + \ln\frac{s}{\mu_F^2}\right),$$
$$\bar{P}'(z,\varepsilon) = \bar{P}(z) + \frac{1}{2}\varepsilon(1-z)^2 + \varepsilon\ln(1-z).$$

Exact analytical integration at NLO

EXACT analytical integration of the NLO MC distributions over the multigluon phase space is again possible(!):

$$\sigma(C_0^{(1)}\Gamma_F\Gamma_B) = \int_0^1 d\hat{x}_F\; d\hat{x}_B\; dz\; D_F(\Xi,\hat{x}_F)\; D_B(\Xi,\hat{x}_B)\; \sigma_B(sz\hat{x}_F\hat{x}_B)$$

$$\times \left\{ \delta_{z=1}(1+\Delta_{S+V}) + C_{2r}^{psMC}(z) \right\}$$

where

$$C_{2r}^{psMC}(z) = \frac{2C_F\alpha_s}{\pi}\left[-\frac{1}{2}(1-z)\right]$$

The above differs from $\overline{MS}$ eq. (90) in Altarelli-Ellis-Martinelli (79)

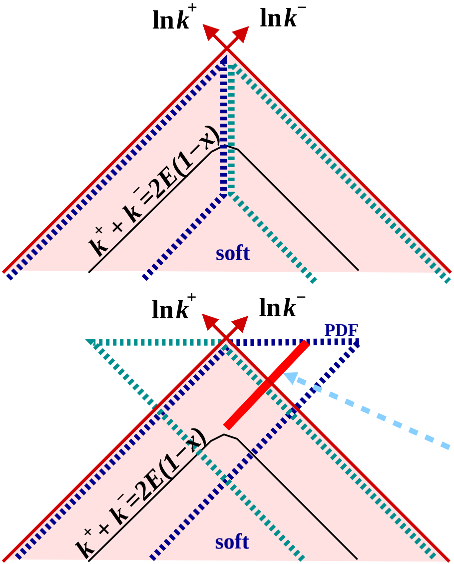
$$C_{2r}^{\overline{MS}}(z) = 2\frac{C_F\alpha_s}{\pi}\left(\frac{\bar{P}(x)}{1-z}\right)_+ [2\ln(1-z) - \ln z]$$

Why? psMC factorization scheme is slightly different from $\overline{MS}$:

$$C_{2r}^{psMC}(z) - C_{2r}^{\overline{MS}}(z) = -2C_{ct}^{psMC}(z) + 2C_{ct}^{\overline{MS}}(z) \simeq 4\frac{C_F\alpha_s}{\pi}\left(\frac{\bar{P}(x)}{1-z}\ln(1-x)\right)_+$$

Difference between $\overline{MS}$ and psMC fact. schemes

Simple kinematics explains $4\ln(1-x)/(1-x)_+$



psMC fact. scheme:

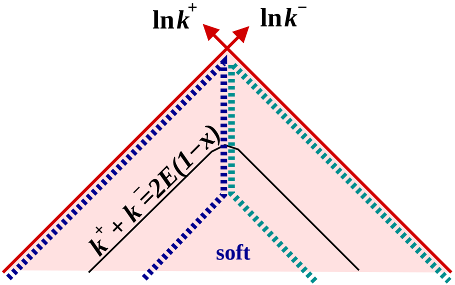
$$0\ \frac{|\ln(1-x)|}{1-x}$$

$\overline{MS}$ fact. scheme:

$$\int_{1-x}^{1/(1-x)} \frac{d\beta}{\beta} = 2\frac{|\ln(1-x)|}{1-x}$$

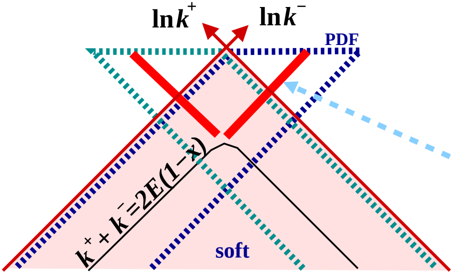
Difference between $\overline{MS}$ and psMC fact. schemes

Simple kinematics explains $4\ln(1-x)/(1-x)_+$



psMC fact. scheme:

$$0\;\frac{|\ln(1-x)|}{1-x}$$



$\overline{MS}$ fact. scheme:

$$\int_{1-x}^{1/(1-x)} \frac{d\beta}{\beta} = 4\,\frac{|\ln(1-x)|}{1-x}$$

Summary on differences with MC@NLO

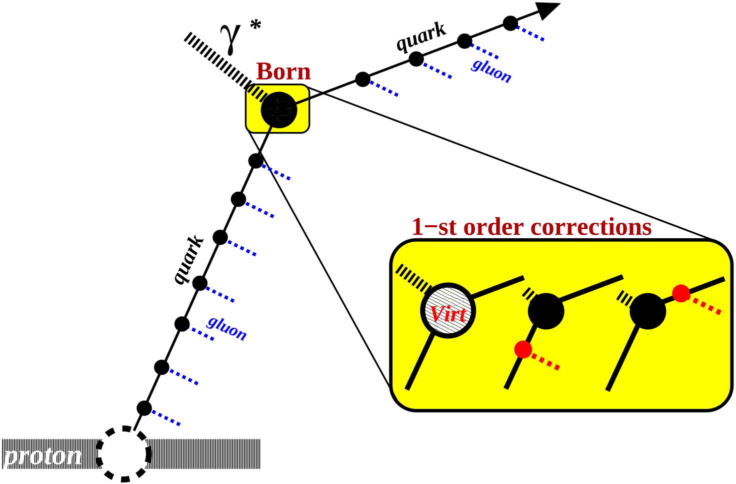
- Very simple and positive MC weight adding NLO on top of LO psMC
- No need to correct for the difference in collinear counterterm of psMC and $\overline{MS}$ schemes.
- Virtual+soft corrections is completely kinematics independent
  – all their annoying $d\Sigma^{c\pm}$ contributions are simply gone!
- Built in resummation of $\frac{\ln^n(1-x)}{1-x}$ terms.
- more transparent relation to CFTs

All the above goodies at the price of:

- rebuilding LO psMC from the scratch
- and slight (but well controlled) change of the factorization scheme in the subtracted hard process ME and in PDFs

NLO corrections to hard process in *ep* DIS

http://jadach.web.cern.ch/jadach/public/CERNjuly2010.pdf describes LO+NLO psMC for DIS process in some detail



Below we only comment on factorization scheme dependence.

NLO corrections to hard process in *ep* DIS

Here in the NLO psMC weight for hard process:

$$\Delta_{V+S} = D^{\overline{MS}}_{DIS}(z) - 2C^{psMC}_{ct}(z) \equiv 0,$$

(exploiting again $\overline{MS}$ calculation of Altarelli+Ellis+Martinelli (1979)), while analytical integration of the multigluon LO+NLO psMC distributions yields the following structure functions:

$$\frac{d\sigma^{NLO}_{DIS}}{dt dx_B} = \frac{2\pi\alpha^2 Q_q^2}{t^2} \int dx dz\; \delta_{x_B=xz}\; D_l(t,x)\Big[W_0(y_B)C_2^{psMC}(z) + 2(1-y_B)z^{-1}C_L(z)\Big],$$

$$C_2^{psMC}(z) = \frac{C_F\alpha_s}{\pi}\left\{-\frac{1+z^2}{2(1-z)}\ln(1-z) - \frac{3}{4}\frac{1}{1-z} + \frac{3}{2} + z\right\}_+$$

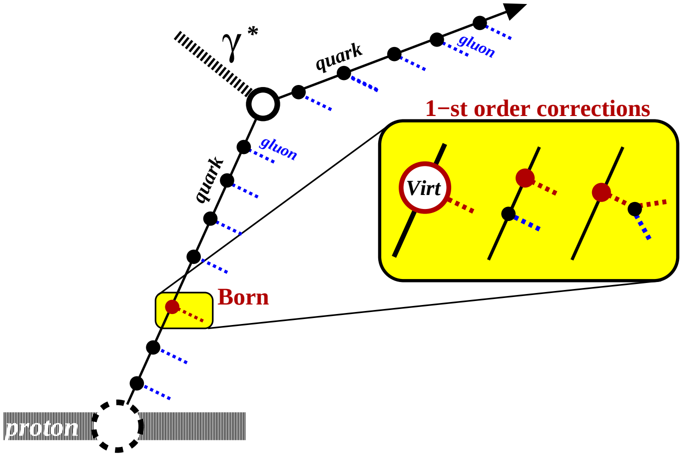
$$C_L(z) = \frac{C_F\alpha_s}{\pi}z^2, \quad W_0(y) = 1+(1-y)^2,\; y_B = t/(sx_B),$$

and x_B is standard Bjorken variable.
The difference with the standard C_2 of $\overline{MS}$ (Bardin+Buras, 78) is entirely due to difference between collinear counterterms of psMC and $\overline{MS}$.

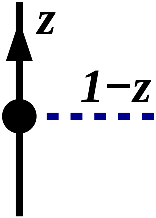
NB Altarelli-Ellis-Martinelli FS-indepenedent difference DY−2*DIS is reproduced by two above psMCs up to NLO exactly!

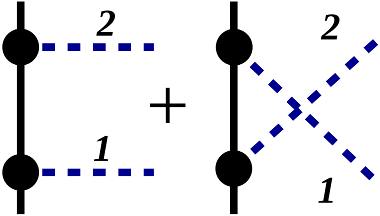
NLO corrections to the ladder parts of psMC

Again starting from Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge), and recalculating their NLO DGLAP kernels.

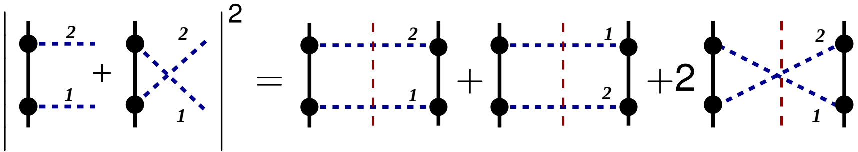


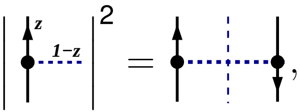
1-st order virtual and real correction (subset) diagrams

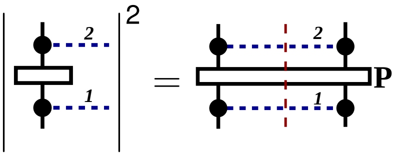
Virtual : $\left(1 + \Delta^{(1)}_{ISR}(z)\right)$ 

Real : 

NOTATION: squared MEs = cut-diagrams, C_F^2 only

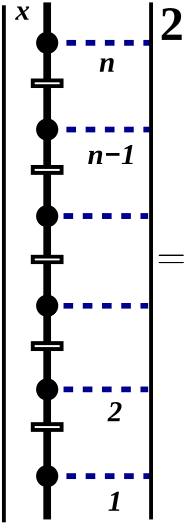






LO ladder = parton shower MC

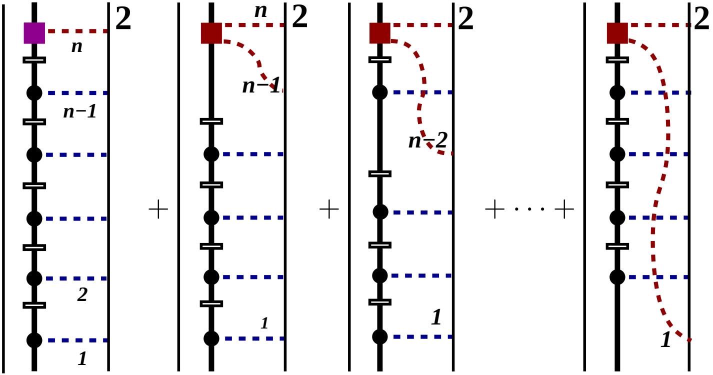
$$\sum_{n=0}^{\infty}$$



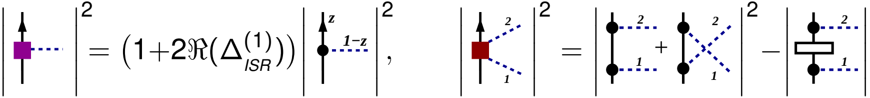
$$= e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0}\, \theta_{Q>a_i>a_{i-1}}\, \rho_{1B}^{(0)}(k_i)\; \delta_{x=\prod z_i}$$

$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}.$$

LO with NLO-corrected kernel at the end of the ladder

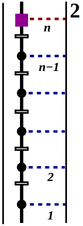


Virt. multiplicative   Undoing LO simplificat.   Sum over trailing LO spectators, essential (BE, YFS61)

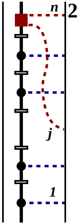


MORE DETAILS:

$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}}$$

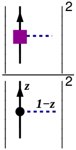


$$+\, e^{-S_{ISR}} \sum_{j=1}^{n-1}$$

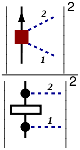


$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \Bigg) \Bigg[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

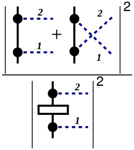
where $d\eta_i = \frac{d^3 k_i}{k_i^0}$, $\beta_0^{(1)} =$



, $W(k_2, k_1) =$



$=$



$- 1.$

Mapping $k_i \to \tilde{k}_i$ instrumental. S_{ISR} = double-log Sudakov, W is non-singular!

Algebraic crosscheck

Analytical integration of NLO part $\sum_j W(\tilde{k}_n, \tilde{k}_j)$ can be done leading to:

$$\sum_{n=1}^{\infty} \int du \int\limits_{Q>a_n>a_{n-1}} \frac{da_n}{a_n} \mathcal{P}_{qq}^{(1)}(u) \left(\prod_{i=1}^{n-1} \int\limits_{a_{i+1}>a_i>a_{i-1}} \frac{da_i}{a_i} \mathcal{P}_{qq}^{(0)}(z_i) \right) \delta_{x=u \prod_{j=1}^{n-1} z_j}.$$

where we recover precisely NLO part (including virtuals) of standard DGLAP kernel $\mathcal{P}_{qq}^{(1)}(u)$ defined according to:

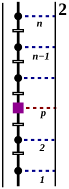
$$\mathcal{P}_{qq}^{(1)}(u) \ln\frac{Q}{q_0} = \int\limits_{Q>a_n>a_0} d^3\eta_n\, \rho_{1B}^{(1)}(k_n)\, \beta_0^{(1)}(z_n)\delta_{u=z_n} + \int\limits_{Q>a_n>a_0} d^3\eta_n \int\limits_{a_n>a_{n'}>0} d^3\eta_{n'}\, \beta_1^{(1)}(\tilde{k}_n, \tilde{k}_{n'})\, \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced, modulo factorization scheme change $\overline{MS} \rightarrow$psMC.

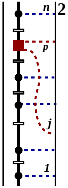
NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p:

$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{ \sum_{p=1}^{n} \Bigg|$$



$$\Bigg|^2 + \sum_{p=1}^{n} \sum_{j=1}^{p-1} \Bigg|$$

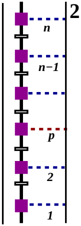


$$\Bigg|^2 \Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

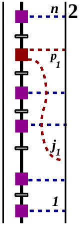
$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

Next step is to add more "NLO insertions",
for instance 2 at positions p_1 and p_2 and sum up over them...
then 3 insertions at p_1, p_2, p_3 and so on
– LO+NLO kernels built up all over along the ladder!

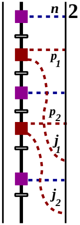
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1}$$



$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1\\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1\\ j_2\neq p_1,j_2}}^{p_2-1}$$



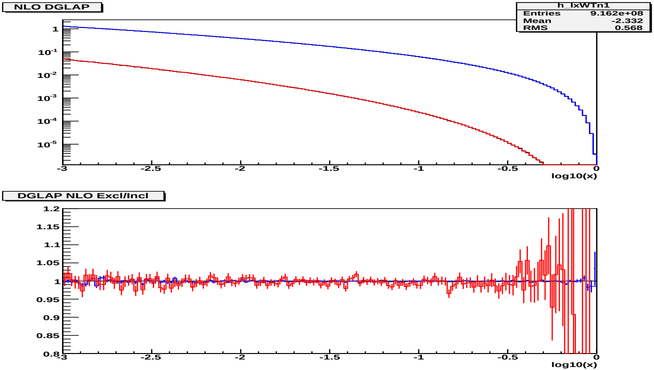
$$\Bigg\}$$

$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \Bigg) \Bigg[1 + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) +$$

$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1\\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1\\ j_2\neq p_1,j_2}}^{p_2-1} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \dots \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.

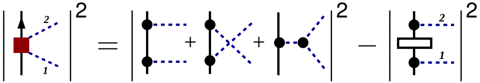
Numerical test of ISR pure C_F^2 NLO MC



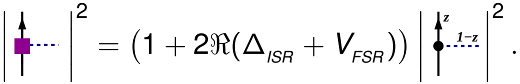
Numerical results for $D(x, Q)$ from inclusive and exclusive **two** Monte Carlos.
Blue curve is single NLO insertion, red curve is double insertion component.
LO+NLO is off scale. Evolution 10GeV$\rightarrow$1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

THE PROBLEM WITH GLUON PAIR COMPONENT OF the NLO KERNEL, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:



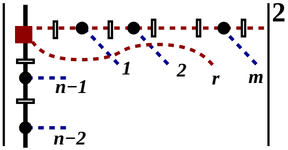
and $-S_{FSR}$ in the virtual correction:



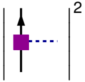
SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

Additional NLO FSR corr. at the end of the ladder:

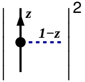
$$e^{-S_{ISR}-S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$



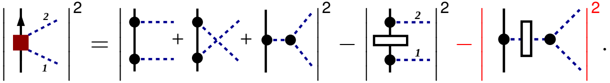
where Sudakov S_{FSR} is subtracted in the virtual part:



$$= \left(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\right)$$



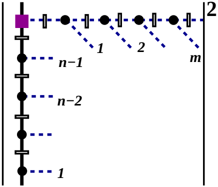
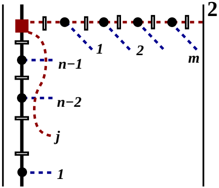
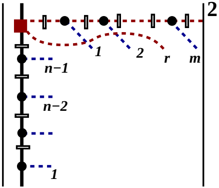
and FSR counterterm is subtracted in the 2-real-gluon part:



The miracle: both are free of any collinear or soft divergence!!!

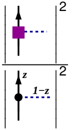
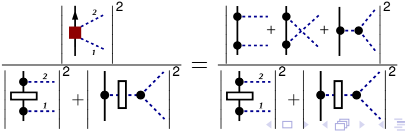
ISR+FSR NLO scheme, NLO corr. at end of the ladder

$\bar{D}_{NS}^{[1]}(x,Q) =$

$e^{-S} \sum_{n,m=0}^{\infty} \Bigg\{$  $+ \sum_{j=1}^{n-1}$  $+ \sum_{r=1}^{m}$  $\Bigg\}$

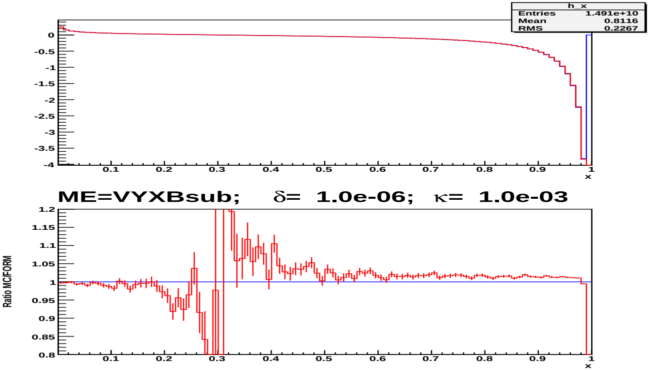
$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \Bigg) e^{-S_{FSR}} \sum_{m=0}^{\infty} \Bigg(\prod_{j=1}^{m} \int_{Q>a_{nj}>a_{n(l-1)}} d^3\eta'_j \; \rho_{1V}^{(1)}(k'_j) \Bigg)$$

$$\times \Bigg[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^{m} W(\tilde{k}_n, \tilde{k}'_r) \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

$\beta_0^{(1)} \equiv$  , $\quad W(k_2,k_1) \equiv$  $- 1.$

3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:



because in this case analytical integration is feasible.
MC agrees precisely with the analytical result.

Summary and Prospects

- New formulation of the collinear factorization, better suited for Monte Carlo implementation is defined.
- NLO contributions to hard process and evolution kernels are already recalculated up to NLO in the new scheme (non-singlet NLO exclusive kernels calculated).
- Implementation in the Monte Carlo is tested at the prototype level; critical MC weights are examined numerically.
- R&D phase (almost) completed, MC realization for W/Z prod. at LHC/Tevatron and DIS proc. becomes main front.