Theoretical calculations for LEP luminosity measurement

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The aims are:

- **to give personal (biased) view of the history of the theoretical calculations of the small angle Bhabha process for LEP experiments**
- **to give account on the "strategical choices" which we did during the 10 years of the struggle for better th. precision**
- **to review briefly the composition of the "theoretical errors" of the luminosity cross section at the end of LEP era.**

Strategic choices done and missed

- **Be sure that all preexisting knowledge is well absorbed and fully exploited.**

  (We absorbed $\mathcal{O}(\alpha)$ MC OLDBAB of Leiden group and older analytical calculations of the QED boxes from italian groups. We profited from Z-gamma of Beenakker.)

- **Be sure that you know precisely what you are after.**

  It was quite tough job to convince us and others that "technical precision" is very important.

  "Very precise" calculations (in principle) on top of huge numerical noise are worth nothing!

- **Build elaborate system of numerical precision "benchmarks", which can be easily recreated periodicaly after every new advancement. Absolutely necessary!**

- **The real art is not to do terribly complicated calculations (and impress everybody with long formulas) but actually avoid them:-)**

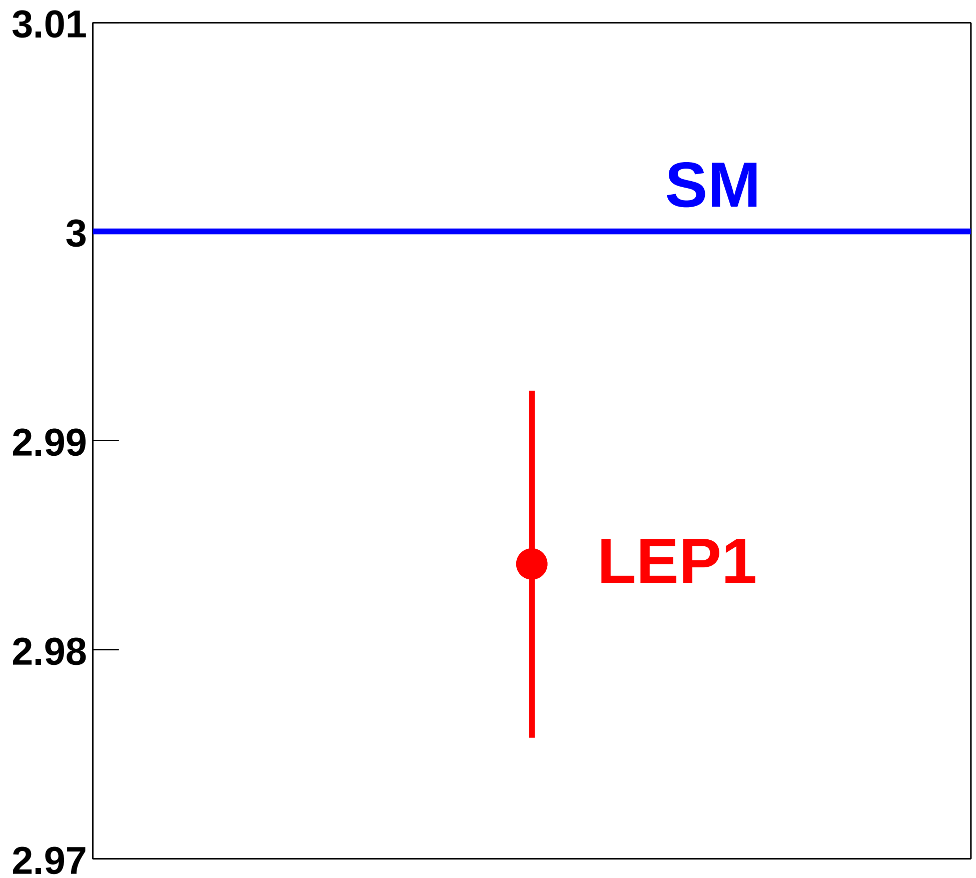
  (Try to calculate/isolate perturbative contributions which are numerically big and skip these which for sure are small. Dont make "eintopf" out of everything!)

- **Good resummations of infrared and collinear contributions to infinite order are worth the same, sometimes more, than adding one more complete fixed order.**

Strategic choices done and missed

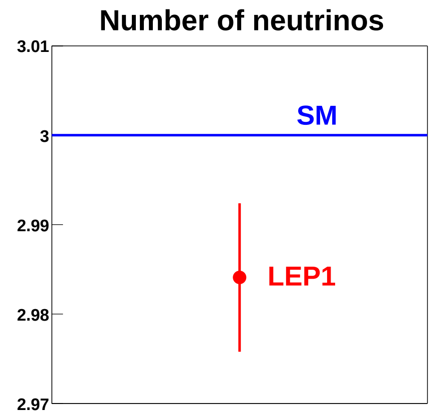
- **Given the limited manpower: Should one put more emphasis on the pure matrix element calculations or on all other things? It has to be well balanced!**
- **Given the limited manpower and aiming at high overall precision for the true realistic complicated experimental event selection is it better to compare:**
  (i) semianalytical calculation versus Monte Carlo program?
  (ii) or rather invest into two different Monte Carlo programs?
  We have opted for (i).
  Looking back I would rather go for (ii), if I started one more time!
- **Sociology: try to get at least one other independent group at the forefront.**
  At the time of 1996 workshop on Bhabha we have finally got an active “ecosystem” of several groups.

Number of neutrinos



$N_\nu = 2.9841 \pm 0.0083$. **It deviates** 1.9σ **from the SM. See ADLO summary paper hep-ex/0112021. Error is dominated by (theoretical) luminosity error** $\delta L/L \simeq 0.05\%$**. OPAL experimental lumi error is a remarkable 0.034%! See EPJ C14 (2000) 373.**

Invisible Z width, number of neutrinos



$$\sigma^{pole}_{ff} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{\Gamma_Z^2} = \frac{12\pi}{m_Z^2} Br(Z\to e^+e^-) Br(Z\to f\bar{f})$$

$$Br(Z\to \mathrm{inv}) = 1 - Br(Z\to \mathrm{had}) - 3Br(Z\to l^+l^-)$$

$$\boxed{R_{\mathrm{inv}} \equiv \frac{Br(Z\to \mathrm{inv})}{Br(Z\to l^+l^-)} = \left(\frac{12\pi}{m_Z^2 \sigma^{pole}_{l^+l^-}}\right)^{1/2} - R_{\mathrm{had}} - 3}$$

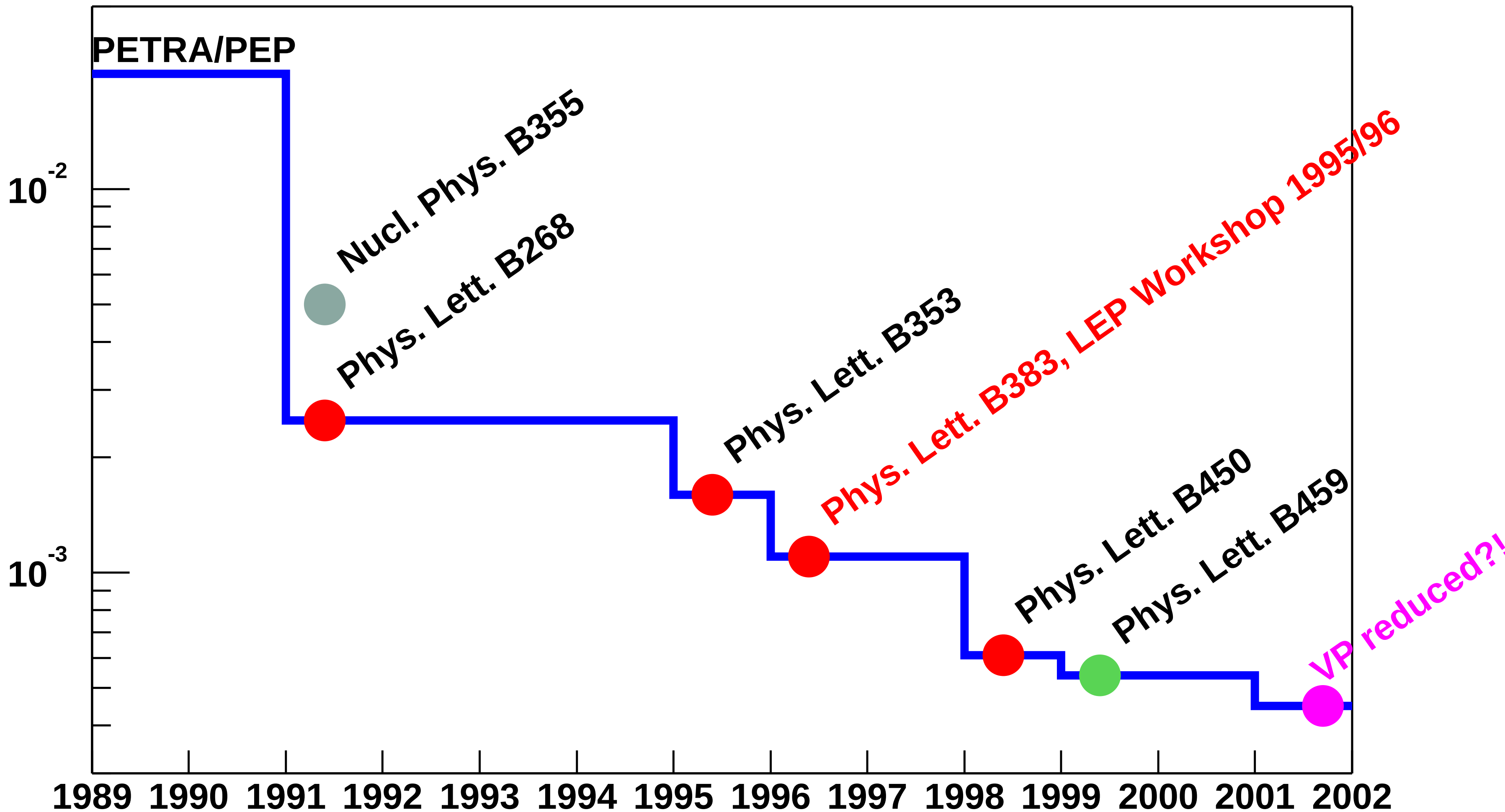
$$R^{\mathrm{SM}}_{\mathrm{inv}} \equiv 3\frac{Br(Z\to\nu\bar{\nu})}{Br(Z\to l^+l^-)} = 5.973 \pm 0.003, \quad \text{for } \alpha_s = 0.119 \pm 0.003$$

$$N_\nu = 3\frac{R_{\mathrm{inv}}}{R^{\mathrm{SM}}_{\mathrm{inv}}} = 2.9841 \pm 0.0083$$

Almost all of the error in N_ν is due to (theoretical) luminosity error entering into $\sigma^{pole}_{l^+l^-}$.

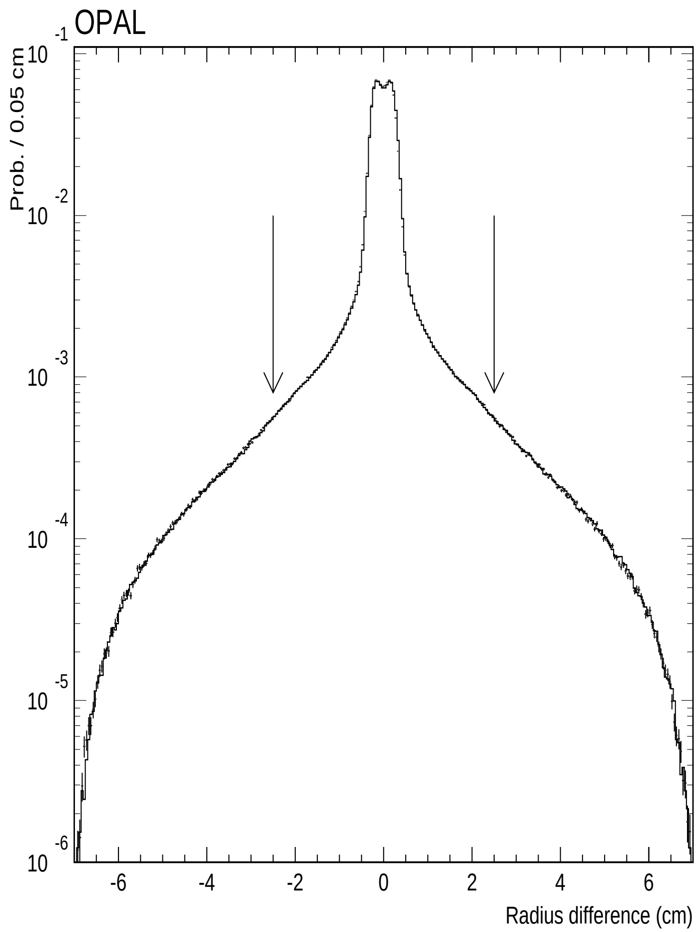
NB. The QED ISR theoretical error $\pm 0.02\%$ in $\sigma^{pole}_{l^+l^-}$ is too small to contribute to N_ν.

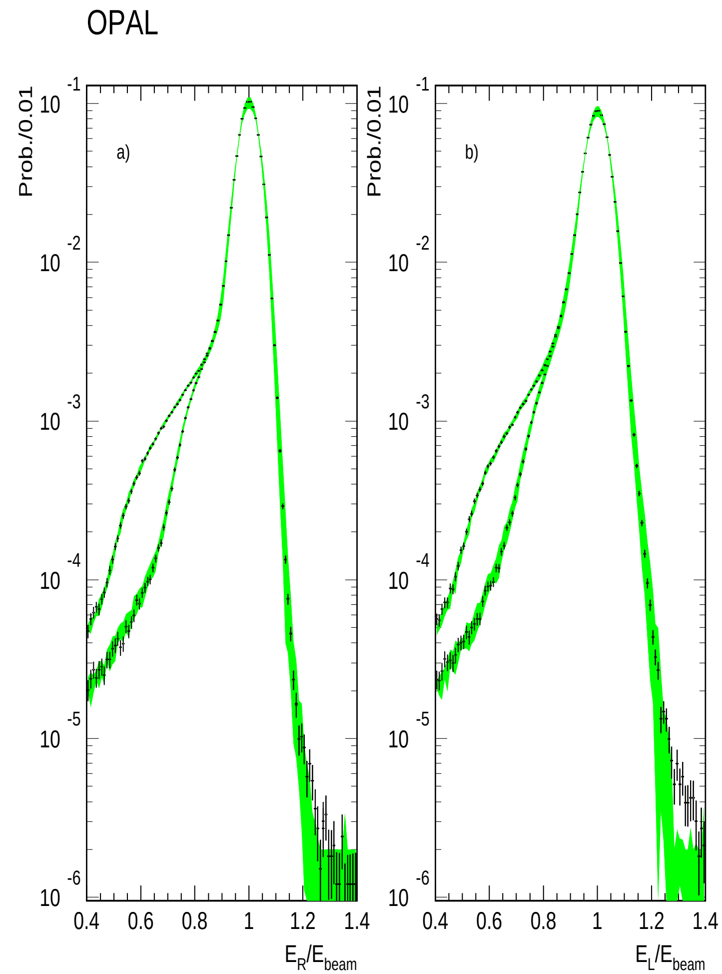
Evolution of luminosity theoretical error at LEP1



- **LEP workshop 1995/96 (0.11%) was "The great consolidation"**
- **Latest published 1999 result: 0.054%**
- **Naive δVP=0.40→0.025% gives 0.044%**

Extraordinary agreement of OPAL data vs. BHLUMI MC, collinearity and energy distrs.





BHLUMI 4.04 Monte Carlo was used by all four LEP experiments. Not only controls luminosity normalization $d\sigma/\sigma = 0.06\%$, but also perfectly agrees with all experimental spectra, with NO "TUNING" to experimental data! (Only one bug in 1995.)

References

Main TH precision improvements marked in red:

[1] W. Beenakker, F. A. Berends and S. C. van der Marck, Nucl. Phys. B **355** (1991) 281.
Photonic $\mathcal{O}(\alpha^2 L_e^2)$ and vacuum polarization (VP)

[2] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **253** (1991) 469.
Technical precision 0.02% establishe for the “baseline” $\mathcal{O}(\alpha^1)$ MC

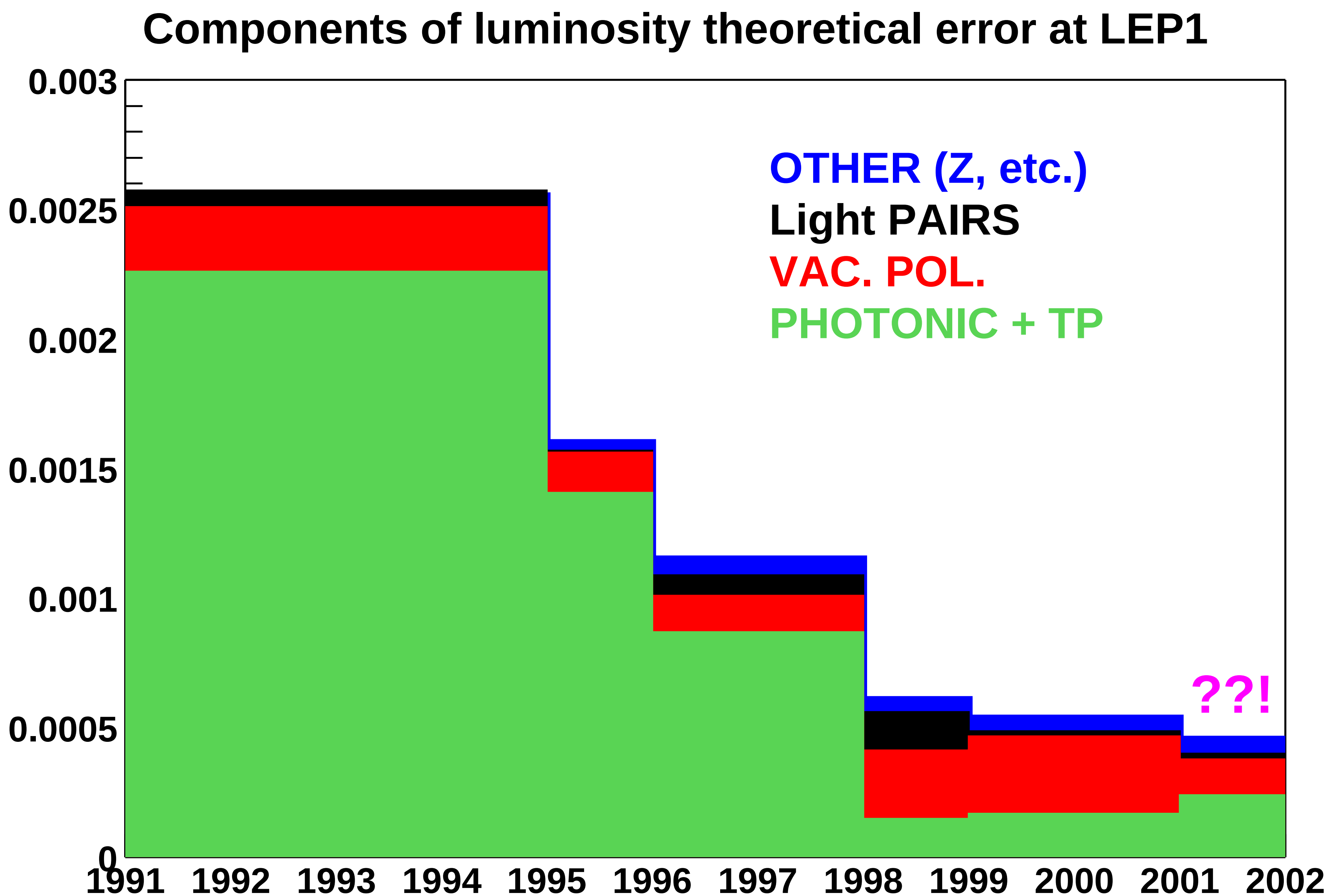
[3] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **260** (1991) 438.
First reliable estimate of the precision of $\mathcal{O}(\alpha^1)_{expon}$ multiphoton BHLUMI MC

[4] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **353** (1995) 362 [Erratum-ibid. B **384** (1996) 488].
Inclusion of $\mathcal{O}(\alpha^2 L_e^2)$ and $\mathcal{O}(\alpha^3 L_e^3)$, new estimate of $\mathcal{O}(\alpha^2 L)$

[5] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238
New estimate of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

[6] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262
New calculation of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

[7] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649
New calculation of missing light real and virtual pairs



Components summed (usually) in the quadrature. Last entry under discussion!

Photonic corrections

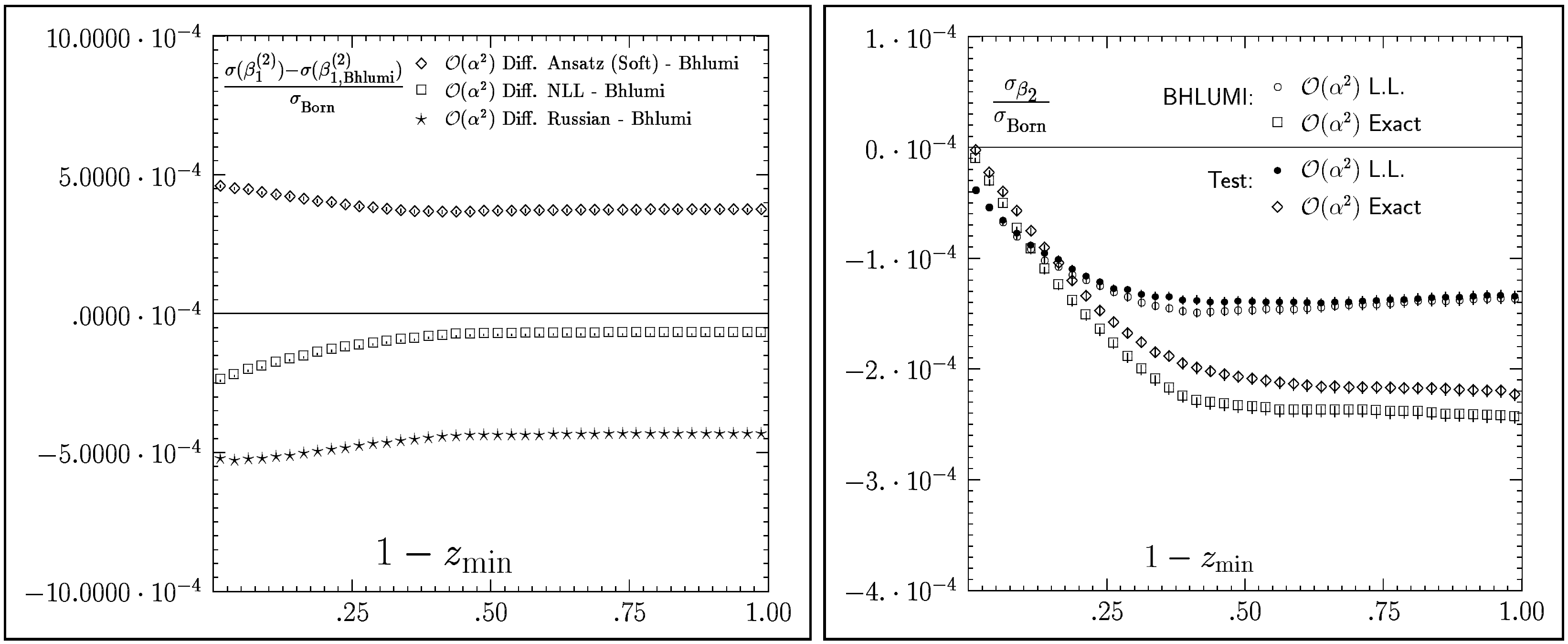
Canonical coefficients in PHOTONIC corrections, $L = \ln(-t_{\rm min}/m_e^2)$					
		$\theta_{min} = 30$ mrad		$\theta_{min} = 60$ mrad	
		LEP1	LEP2	LEP1	LEP2
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi}4L$	137×10^{-3}	152×10^{-3}	150×10^{-3}	165×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2}\frac{\alpha}{\pi}$	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}
$\mathcal{O}(\alpha^2 L^2)$	$\frac{1}{2}\left(\frac{\alpha}{\pi}4L\right)^2$	9.4×10^{-3}	11×10^{-3}	11×10^{-3}	14×10^{-3}
$\mathcal{O}(\alpha^2 L)$	$\frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}4L\right)$	0.31×10^{-3}	0.35×10^{-3}	0.35×10^{-3}	0.38×10^{-3}
$\mathcal{O}(\alpha^3 L^3)$	$\frac{1}{3!}\left(\frac{\alpha}{\pi}4L\right)^3$	0.42×10^{-3}	0.58×10^{-3}	0.57×10^{-3}	0.74×10^{-3}

Anticipated size of QED photonic corrections in small angle Bhabha at LEP.

Already in 1992 we have anticipated MISSING photonic $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^3)$ in BHLUMI 4.x to be 0.1% or less. It took some time to prove it.

Photonic corrections $\mathcal{O}(\alpha^2 L)$

Very big reduction of photonic $\mathcal{O}(\alpha^2 L)$ error, 0.10% $\longrightarrow$ 0.031%, was done in ref. [1]



Missing $1\gamma_{real} \otimes 1\gamma_{virt.}$ and $2\gamma_{real}$

The easiest $2\gamma_{virt.}$ was also calculated. All the above with negligible technical error!

Total missing photonic $\mathcal{O}(\alpha^2 L)$ was found 0.031% for typical LEP1 experiment

Unfortunately, this $\mathcal{O}(\alpha^2 L)$ stays outside BHLUMI 4.04 used in the experiments.

[1] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262

Photonic corrections $\mathcal{O}(\alpha^3 L^3)$

Missing $\mathcal{O}(\alpha^3 L^3)$ in BHLUMI was estimated $< 0.010\%$ in 1992.

In ref.[1] missing $\mathcal{O}(\alpha^3 L^3)$ was calculated to be ≤ 0.015 for typical LEP1 detector.

The correction is unfortunately not in the main multiphoton BHLUMI 4.x code.

These corrections would be relatively easy to complete in the multiphoton BHLUMI 4.x,

similarly as in the similar photonic matrix element in KORALW and KKMC.

We have also found in ref.[1] indication that $\mathcal{O}(\alpha^2)$ calculation without exponentiation will necessarily miss $\sim 0.030\%$ worth of $\mathcal{O}(\alpha^3 L^3)$.

Hence unexponentiated (fixed order) $\mathcal{O}(\alpha^2)$ would not help much LEP1 luminosity measurement. Memento!!!

[1] S. Jadach and B. F. Ward, Phys. Lett. B **389** (1996) 129

Real and virtual light fermion pairs $f\bar{f}, f = e, \mu, q$

Main contribution is from e^+e^- pairs.

For typical calorimetric LEP detector there are strong virtual-real cancellations.

Contrary to early claims (by Italian and Russian groups) that this contribution is huge $\sim 0.500\%$, in ref. [1] the light $f\bar{f}$ contrib. was found $\boxed{-0.013\% \pm 0.020\%}$ for realistic event selection, hence $\boxed{0.030\%}$ was used in 1996 as a component of theoretical error.

Using similar technique this contrib. was calculated independently in refs. [2-3], where for typical LEP acceptance it was found $\boxed{\text{from } -.025\% \text{ to } -.030\%}$ with technical+physical precision $\boxed{\simeq 0.010\%}$.

Both groups provided MC tools for correcting experimental data.

[1] S. Jadach, M. Skrzypek and B.F.L. Ward Phys. Rev. **D55** (1997) 1206

[2] G.Montagna, M.Moretti, O.Nicrosini, A.Pallavicini, F.Piccinini, Nucl. Phys. B **547** (1999) 39

[3] G.Montagna, M.Moretti, O.Nicrosini, A.Pallavicini, F.Piccinini, Phys. Lett. B **459** (1999) 649

Technical precision to be never forgotten! Depends on event selection!

In ref. [1] we determined technical precision of the $\mathcal{O}(\alpha)$ (no exponentiation) MC calculation to be $\boxed{\text{TP}=0.013\% \pm 0.017\%}$ = diff. between MC and semi-analytical codes $\pm$ MC statistical error.

Since ref. [1] we believe into technical precision $\boxed{\text{TP}\simeq 0.020\% - 0.030\%}$ **for the bulk of the** $\mathcal{O}(\alpha)$ **MC small angle Bhabha cross section!**

New test at LEP workshop, see Fig. 15 in ref. [2], for **realistic** event selection provided $\boxed{\text{TP}\simeq 0.030\%}$ using difference of two independent $\mathcal{O}(\alpha)$ MC programs (no expon.).

For multiphoton BHLUMI TP was incorporated into the photonic TH error 0.10%, which was estimated in ref. [2] examining variety of the MC programs. It was *guessed* $\simeq 0.040\%$ in Table 21 in ref. [2].

In ref. [3] for "quasi-realistic" event selection the comparison of the multiphoton BHLUMI with special high quality semi-analytical calculation has led to $\boxed{\text{TP}\simeq \pm 0.017\%}$

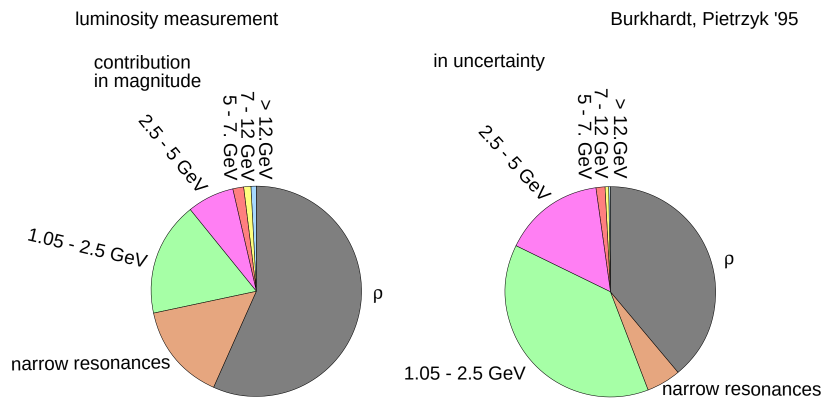
Summarizing $\boxed{\text{TP}\simeq \pm 0.030\%}$ **seems to be a reasonable estimate of the technical precision for the multiphoton BHLUMI 4.x and realistic LEP detectors.**

[1] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, "Analytical $\mathcal{O}(\alpha)$ Distributions For Bhabha Scattering At Low Angles," Phys. Lett. B **253** (1991) 469.

[2] S. Jadach *et al.*, "Event Generators for Bhabha Scattering,", in CERN Yellow Report CERN-96-01, arXiv:hep-ph/9602393.

[3] S. Jadach and B. F. Ward, "Semi-analytical third-order calculations of the small-angle Bhabha cross sections," Acta Phys. Polon. B **28** (1997) 1907.

Vacuum polarization



Since 1996-98 refs. [1-2] are used to provide hadronic vacuum corrections to small angle Bhabha.

The error due to VP was $\boxed{0.040\%}$, its relative importance increased over the years.

As seen from "pie-plot" above it comes mainly from $R(s)$ in the rho region and above.

There, recent measurements of $R(s)$ are better and will be even better. Can we profit?

At $\langle\theta\rangle = 0.034$, ie. $\langle\sqrt{|t|}\rangle = 1.54$GeV, using refs. [1] and [3] we have:

$\Re\Pi_{1995} = 0.541 \pm 0.022$ and $\Re\Pi_{2001} = 0.535 \pm 0.014$, (H. Burkhardt, private communication).

Rescaling naively we get reduction of error due to VP: $\boxed{0.040\% \rightarrow 0.025\%}$

[1] H. Burkhardt and B. Pietrzyk, Phys. Lett. B **356**, 398 (1995).

[2] S. Eidelman and F. Jegerlehner, Z. Phys. C **67**, 585 (1995) [arXiv:hep-ph/9502298].

[3] D. Karlen and H. Burkhardt, Eur. Phys. J. C **22**, 39 (2001) [arXiv:hep-ex/0105065].

Error budget at LEP Workshop 95/96, “The Great Consolidation” (in red)

	LEP1		LEP2
Type of correction/error	Ref.[1]	Ref. [2]	Ref.[2]
(a) Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	0.10%	0.20%
(a) Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.008%	0.015%	0.03%
(c) Vacuum polarization	0.05%	0.04%	0.10%
(d) Light pairs	0.04%	0.03%	0.05%
(e) Z-exchange	0.03%	0.015%	0.0%
Total	0.16%	0.11%	0.25%

[1] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **353** (1995) 362 [Erratum-ibid. B **384** (1996) 488].

[2] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238

My personal update of LEP1 theoretical error, Febr. 2003 (red/magenta)

Type of correction/error	Ref.[1]	Ref. [2]	Ref. [3]	My update
Technical precision	—	(0.030%)	(0.030%)	0.030%
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027%	0.027%	0.027%
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.015%	0.015%
Vacuum polarization	0.04%	0.04%	0.040%	0.025%
Light pairs	0.03%	0.03%	0.010%	0.010%
Z-exchange	0.015%	0.015%	0.015%	0.015%
Total	0.11%	0.061% (0.068)	0.054% (0.061)	0.53%

[1] A. Arbuzov *et al.* *LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238

[2] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262

[3] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649

Summary

- **Summarizing, the present theoretical error of low angle Bhabha $\simeq$0.05%-0.07% seems rather solid.**
- **The room for an easy improvement exists (vacuum polarization).**
- **It looks that the ”technical precision” was slightly underestimated. When corrected, almost cancels improvement due to VP.**
- **Radical improvement of the TH precision to the level of $\leq 0.020\%$, i.e. below the best experimental error 0.034% is feasible.**
- **This would require first of all reduction of the technical precision and once again of the photonic QED, including Z-exchange.
(VP will get reduced another factor 2 in the meantime.)**
- **Somehow we made a “full circle”, and once again the technical precision and the photonic corrections “are the king”.**