Matrix-Element Based Parton Shower MC
KRKMC Project

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The name of the game

Can we construct Parton Shower Monte Carlo for QCD ISR:

- **based firmly on Feynman Diagrams (ME) and LIPS,**
- **based rigorously on the collinear factorization (EGMPR, CSS),**
- **implementing strictly NLO $\overline{MS}$ DGLAP evolution ?**

The aim of this talk is to show that YES, we can!

And to show first numerical implementation – proof of the concept.

The Framework: Two Monte Carlos and Analytical calculation

- Markovian MC with standard integrated LO+NLO DGLAP kernel (gluonstrahlung part)
- Markovian MC with UNINTEGRATED LO+NLO DGLAP kernel. NEW!!!
- Analytical integration leading to NLO DGLAP kernel following Curci-Furmanski-Petronzio (1980); Feynman diagrams $\times$ LIPS, see previous talk by MS.

Introduction: simple Markovian LO parton shower MC

MMCtL = Simple Markovian LO parton shower MC with:

- Transverse momentum $q = e^t = k^T$ as evolution variable
- Mapping of q_i, x_i and φ_i into 4-momenta k_i^μ in the standard LIPS:
  $k_j^T = e^{t_i},\ k_j^- = 2(x_{j-1} - x_j)E_h,\ k_j^+ = (k_j^T)^2/k_j^-.$
- LO DGLAP evol. kernel $\mathcal{P}^L(z)$ for pure gluonstrahlung.

$$D^L(t,x) = Te^{\int_{t_0}^t dt\ \alpha' \otimes \mathcal{P}^L}(x), \quad \mathcal{P}^L(z) = \delta_{z=1} \ln\delta + \theta_{1-z>\delta} \frac{1+z^2}{2(1-z)},$$

$$D_0^L(t,x) = e^{-S}\delta(1-x),$$

$$D_1^L(t,x) = e^{-S} \int_{t_0}^t dt_1 \int_0^{2\pi} \frac{d\phi_1}{2\pi}\ \alpha' \mathcal{P}_\theta^L(x),$$

$$D_2^L(t,x) = e^{-S} \int_{t_0}^t dt_2 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_{t_0}^{t_1} dt_1 \int_0^{2\pi} \frac{d\phi_2}{2\pi}\ \alpha'^2\ \mathcal{P}_\theta^L \otimes \mathcal{P}_\theta^L(x),$$

$$D_3^L(t,x) = \dots$$

$$S = (t-t_0)\alpha' \left(\ln\frac{1}{\delta} + \frac{3}{4}\right), \quad (f\otimes g)(x) \equiv \int dydz\delta_{x=yz} f(y)g(z), \quad \alpha' = \frac{C_F\alpha_S}{\pi}.$$

First MC with standard integrated LO+NLO kernel

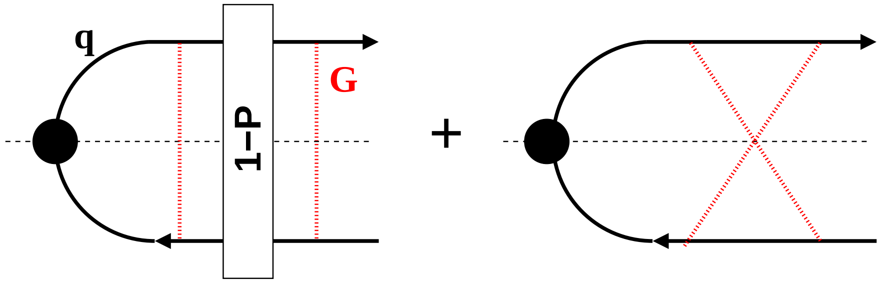
MMCtLN = Markovian with N+L integrated kernel:

- As before, $k^T =$ evolution variable + mapping into standard LIPS,
- But L+N kernel $\mathcal{P}^{L+N}(\alpha', z) = \mathcal{P}^L(z) + \alpha' \mathcal{P}^N(z)$, gluonstrahlung.
- NLO part $\mathcal{P}^N(z)$ is the integrated/inclusive DGLAP kernel, C_F-part.

$$D_1^{L+O}(t,x) = e^{-S} \int_{t_0}^{t} dt_1 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \left(\alpha' \mathcal{P}_\theta^L(x) + {\alpha'}^2 \mathcal{P}^N(x)\right),$$

$$D_2^{L+O}(t,x) = e^{-S} \int_{t_0}^{t} dt_2 \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_{t_0}^{t_1} dt_1 \int_0^{2\pi} \frac{d\phi_2}{2\pi} \, {\alpha'}^2 \, \mathcal{P}_\theta^{L+N} \otimes \mathcal{P}_\theta^{L+N}(x),$$

$\mathcal{P}^N(\alpha', z)$ we have re-calculated from two Feynman diagrams (only real)



following methodology of Curci+Furmanski+Petronzio (1980):

$$\mathcal{P}^N = \frac{1+3x^2}{16(1-x)} \ln^2(x) + \frac{2-x}{4} \ln(x) + \frac{3}{8}(1-x).$$

Second MC with Unintegrated LO+NLO kernel

MMCtLNU = Markovian with N+L Unintegrated kernel:

- As before k^T as evolution variable and mapping into standard LIPS,
- But NLO part of evol. DGLAP kernel kept in UNINTEGRATED form, all over the 2-gluon LIPS!

$$D_1^{L+N}(t,x) = e^{-S} \int_{t_0}^{t} dt_1 \int_0^{2\pi} \frac{d\phi_1}{2\pi}\; \alpha' \mathcal{P}_\theta^L(x),$$

$$D_2^{L+N}(t,x) = e^{-S} \int_{e^{t_0}}^{e^t} \frac{dk_2^T}{k_2^T} \int_0^{2\pi} \frac{d\phi_2}{2\pi} \int_{e^{t_0}}^{k_2^T} \frac{dk_1^T}{k_1^T} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$
$$\int_{x=z_1 z_2} dz_1 dz_2\; {\alpha'}^2 \; \left[\mathcal{P}^L(z_1)\mathcal{P}^L(z_2) + D_2^N(k_1^\mu, k_2^\mu)\right]$$

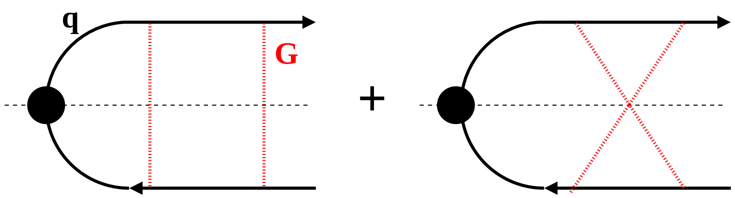
where

$$\rho_2^{L+N}(k_1^\mu, k_2^\mu) = \frac{(k_2^T)^2}{8(k_1^T)^2}\left[\mathcal{P}^L(z_1)\mathcal{P}^L(z_2) + D_2^N(k_1^\mu, k_2^\mu)\right]$$

comes directly from Feynman diagrams $\times$ LIPS.

Hence **MMCtLNU** is (Matrix Element) **ME-based**.

ρ_2^{L+N} from Feynman diagrams



Ladder uncrossed diagram:

$$\int d\text{LIPS}\ \rho_2^{L+N} = \mathcal{N} C_F \int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\delta_{1-x=\alpha_1+\alpha_2} \int d^{2+2\epsilon}\mathbf{k}_1 d^{2+2\epsilon}\mathbf{k}_2\ \mu^{-4\epsilon}$$
$$\times\ \Theta_{Q^2>\max(\mathbf{k}_1^2,\mathbf{k}_2^2)} \left\{ \frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} + \frac{T_2(\alpha_1,\alpha_2,\epsilon)}{\alpha_2^2}\frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} + \frac{T_3(\alpha_1,\alpha_2)}{\alpha_2}\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2} \right\} \frac{1}{q^4(k_1,k_2)}$$

Ladder crossed (interference) diagram:

$$\int d\text{LIPS}\ \rho_2^{L+N} = \mathcal{N} C_F \int \frac{d\alpha_1}{\alpha_1}\frac{d\alpha_2}{\alpha_2}\delta_{1-x=\alpha_1+\alpha_2} \int d^{2+2\epsilon}\mathbf{k}_1 d^{2+2\epsilon}\mathbf{k}_2\ \mu^{-4\epsilon}\ \theta_{\max(\mathbf{k}_1^2,\mathbf{k}_2^2)\le Q^2}\ \frac{1}{q^4(k_1,k_2)}$$
$$\times \left\{ \frac{2T_1^x(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} - T_{2a}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_1\mathbf{k}_2^2} - T_{2b}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_2\mathbf{k}_1^2} + T_3^x(\alpha_1,\alpha_2)\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{\mathbf{k}_1^2\mathbf{k}_2^2} \right\}$$

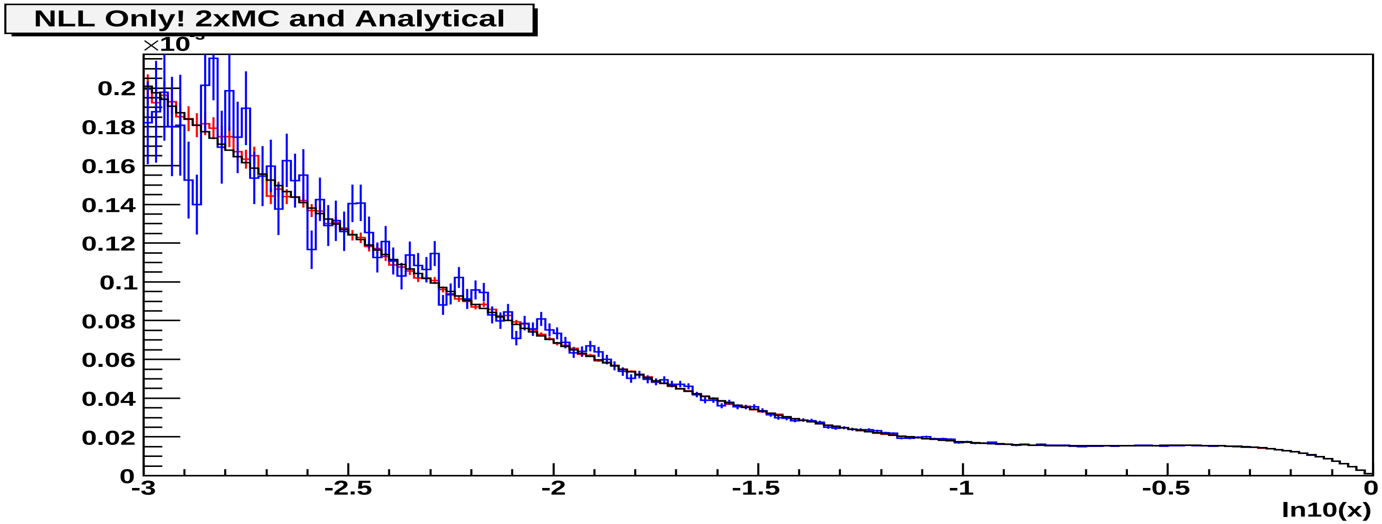
where T_i and T_i^x are γ-trace factors.

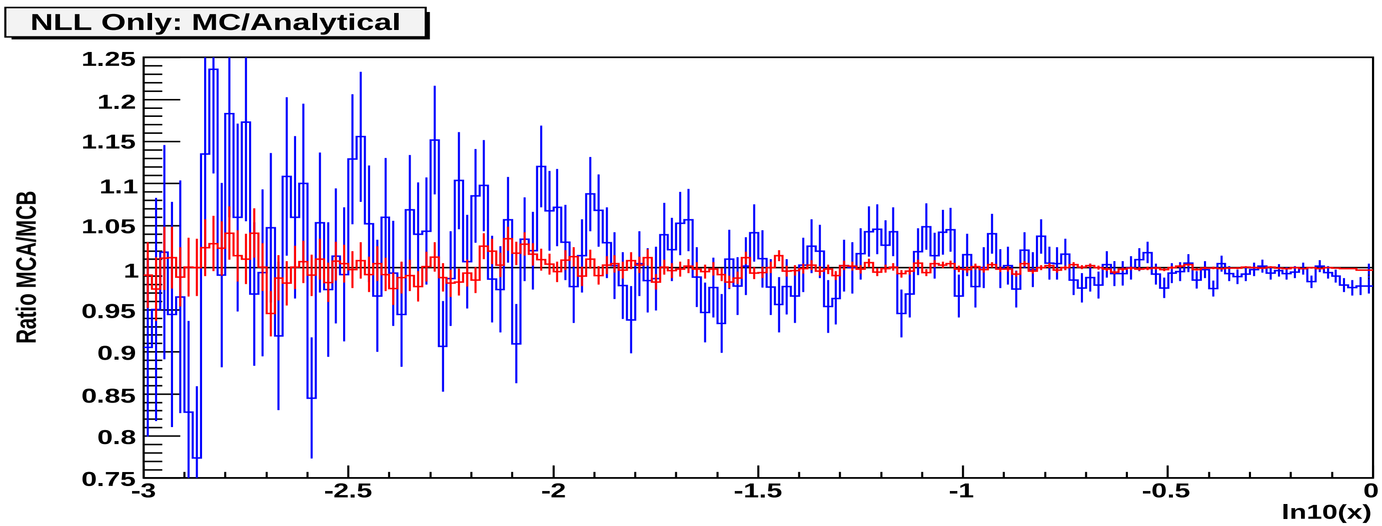
The term $\frac{\mathcal{P}^L(z_1)\mathcal{P}^L(z_2)}{(k_1^T)^2}$ is identified with the "soft counterterm" $\frac{T_2(\alpha_1,\alpha_2,0)}{(k_1^T)^2 q^4(0,k_2)}$.

The above ME from small FORM program, up 10 γ's, gluons in the axial gauge.

Curci-Furmanski-Petronzio spin projectors, dimensional reg. etc.

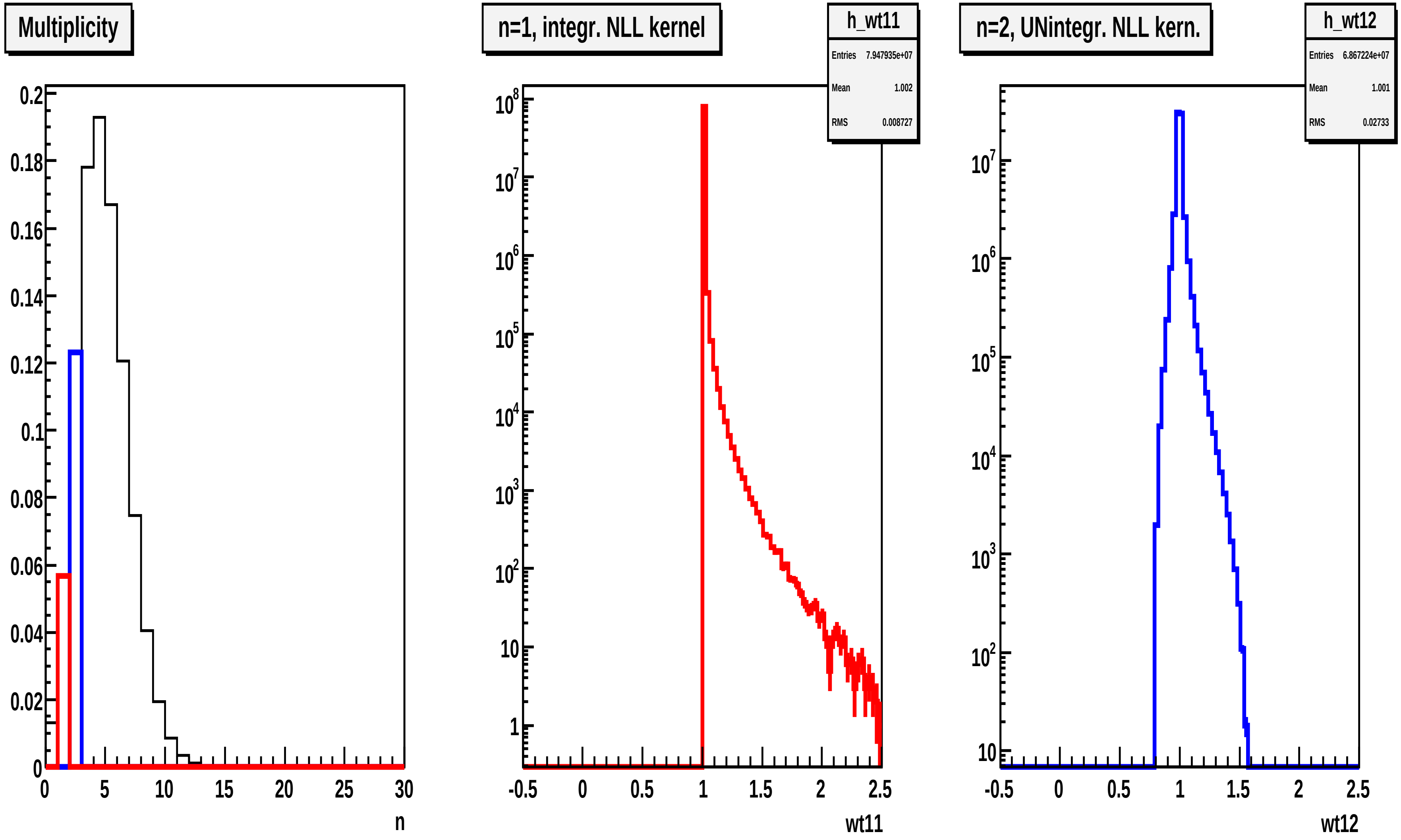
Our principal result from 2 MCs and Analytical integration





- **BLACK** Analytical formula from Curci-Furmanski-Petronzio (table 1)
- **RED** Integrated kernel in Markovian MC (MMCtLN), n=1!
- **BLUE** UNINTEGRATED kernel in Markovian MC (MMCtLNU), n=2!

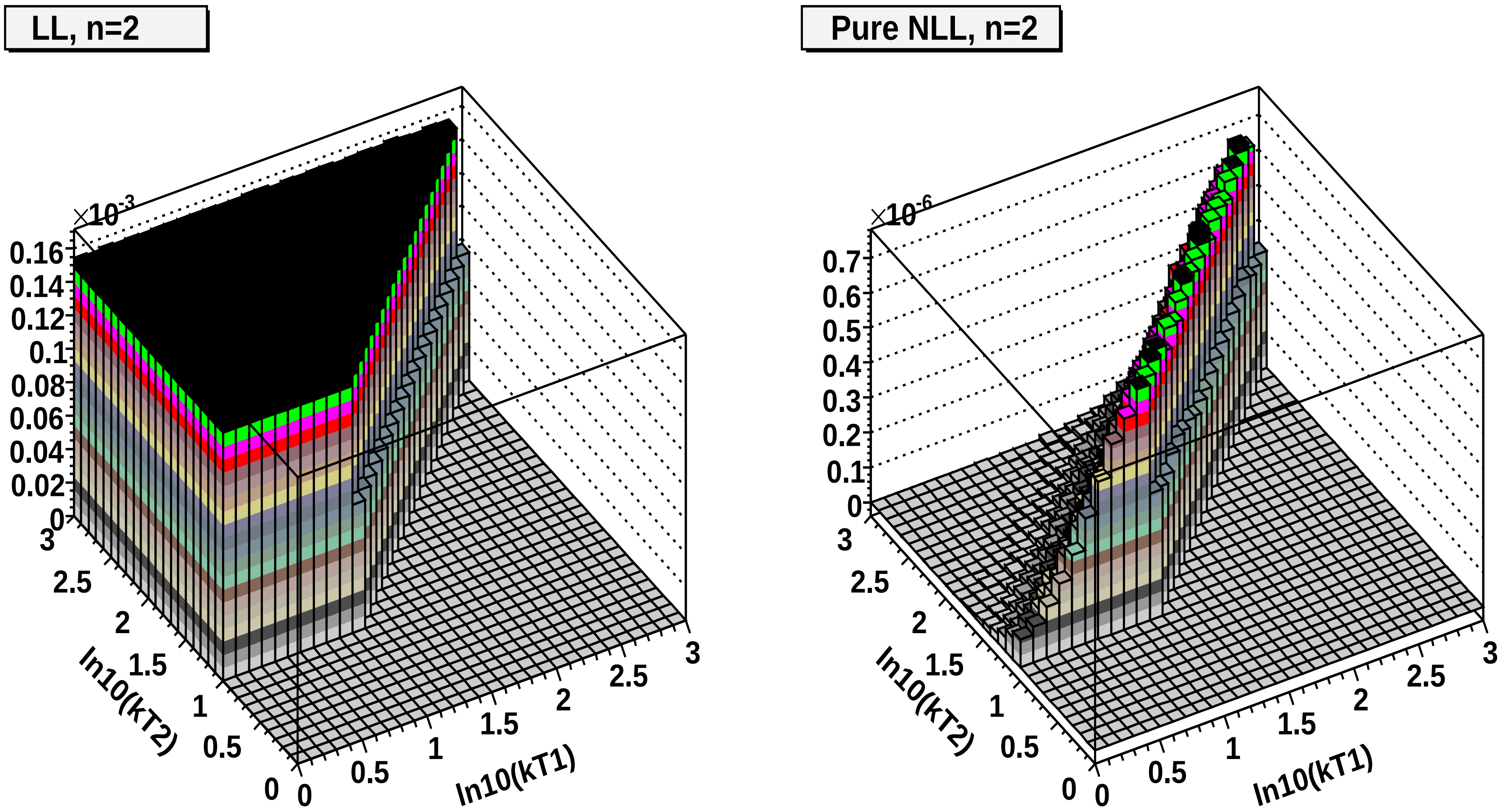
Weight distributions for 2 types of NLO in the MC



Unintegrated kernel (blue) yields more regular weight distribution!

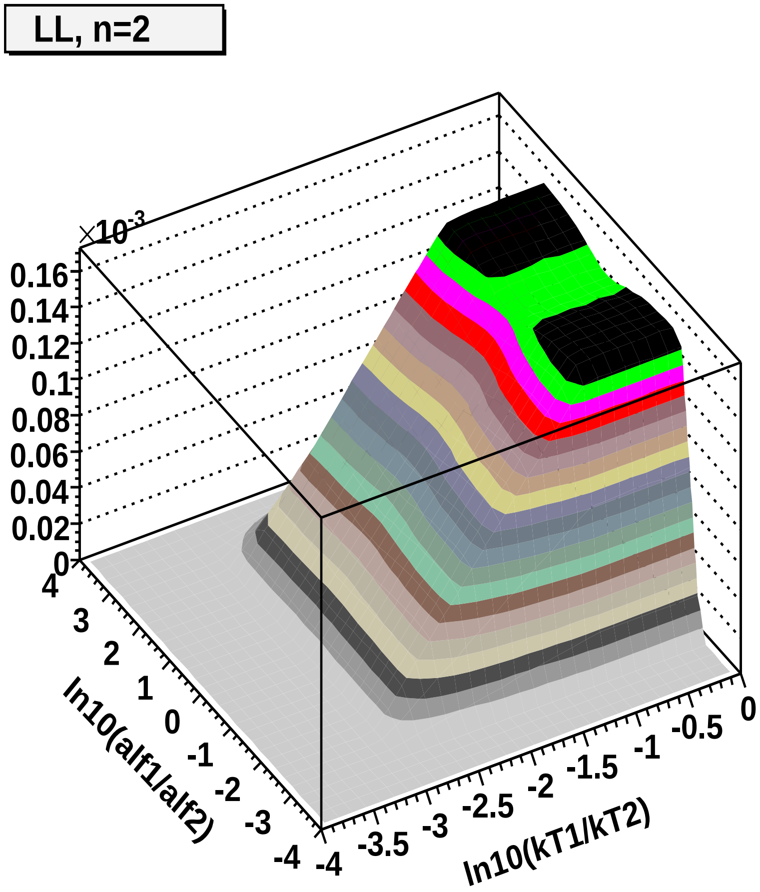
Weight is positive and has small variance.

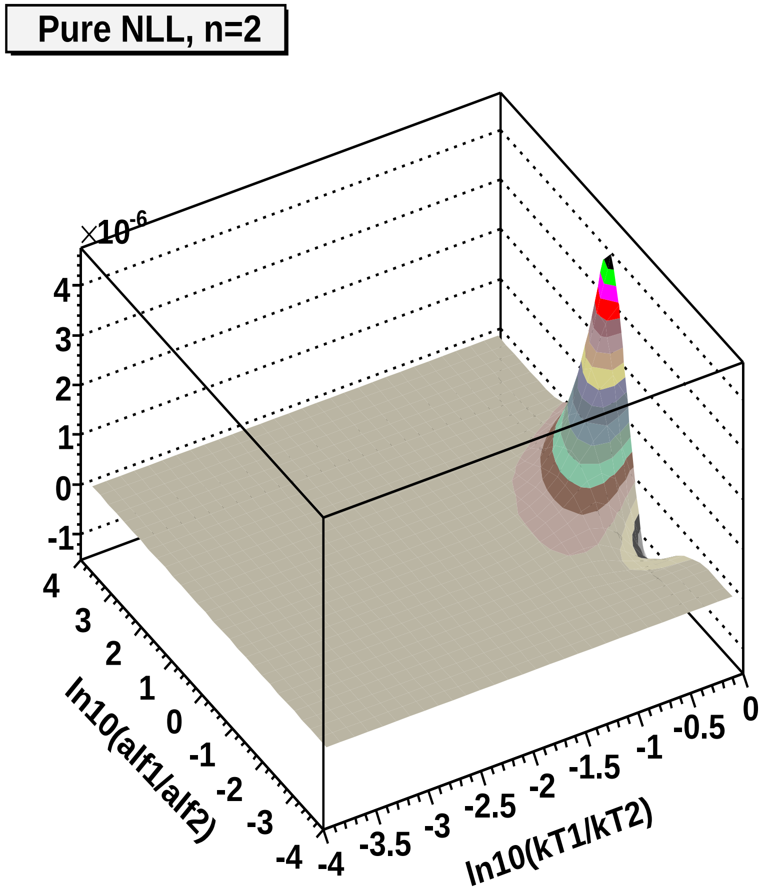
More details: $n = 2$ L+N MC distr. of kT1 vs. kT2



Unintegrated kernel requires $Q > k_2^T > k_1^T > 0$ in order to reproduce CFP formula.
This is why in the MC test we stay within the limits $1TeV > k_2^T > k_1^T > 1GeV$, with the additional condition $k_2^T > 10GeV$, see above left plot.
In the right plot we see more clearly why so.
Right plot also shows how do we loose one big coll. log in the NLO with respect to LO.

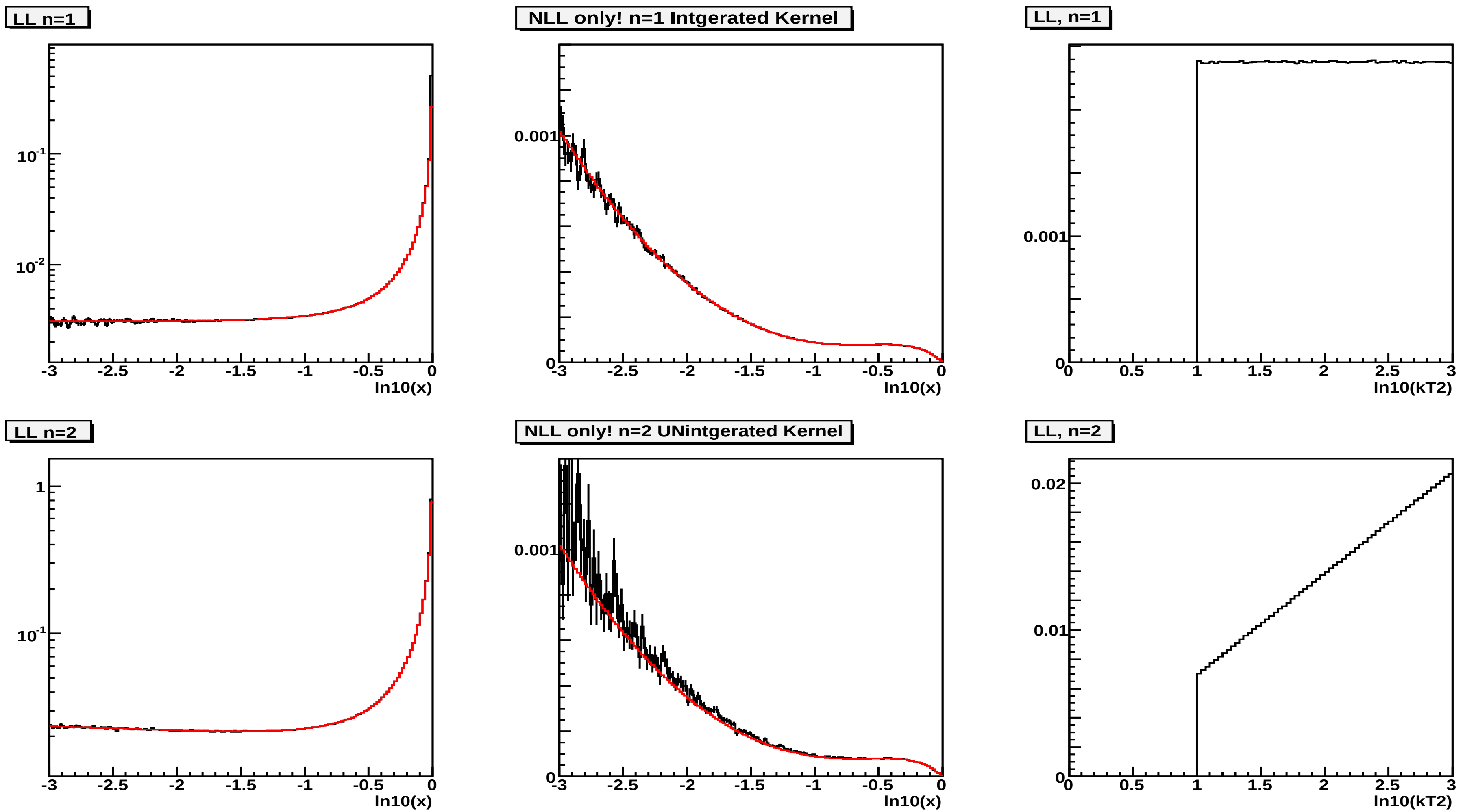
Pure NLO cortib. from MC with UNINTEGRATED kernel, n=2





RHS plot above demostrates how NLO unintegrated kernel act as an "short range correlator" in the combined space of k^T and lightcone variables of the emitted particles. ($\alpha_2 = x - x_1, \alpha_1 = x_1 - x_0$.)
We reproduce the same plot as in the previous Skrzypek's talk within the PS MC.

Picture galery from 2 MCs and analytical



Left plots: LO distributions for n=1,2 from MC (black) and analytical form. (red).

Middle plots: once again NLO from MC with integrated (n=1) and unintegrated kernels (n=2) and analytical formula (red).

Right plots: distribution of the bigest k^T for $n = 1, 2$.

Relation of KRKMC to other schemes/projects

- **NLO DGLAP:**
  4-digit compatibility with it as a test, callibration tool for MC
- **Fixed Order QCD Calculus:**
  similarities in methodology of removing ϵ_{IR} (soft counterterm)
- **YFS:**
  Similiarities in constructing IR-finite β's; Also similar methodology of re-inserting β's in the MC encapsulating IR singularities
- **MCatNLO:**
  KRKMC is NLO parton shower MC written from the scratch, not a "piggy-back" on top of any classic PSMC.
- **Backward Evolution:**
  Obsolete, CMC (Constrained MC) algorithm will be used.
- **Standard collinear PDFs:**
  Will not be used. However, to be reproduced numericaly as a test.

Discussion, Conclusions

- Unintegrated NLO kernel within full 2-particle LIPS in the MC can be constructed.
- Dimensional regularization can be removed.
- The integrand of the NLO kernel features nice IR cancellations, such that only **short range correlation** remain for large y_i and α_i. No long tails! No cancellations between distant regions in the LIPS!
- Re-insertion of the NLO unintegrated kernel into LO MC model representing LO+NLO (DGLAP) evolution looks perfectly feasible.
- Monte Carlo weight looks regular/positive.
- A decisive/critical milestone towards NLO parton shower MC has been reached.

What next?

Immediate plans:

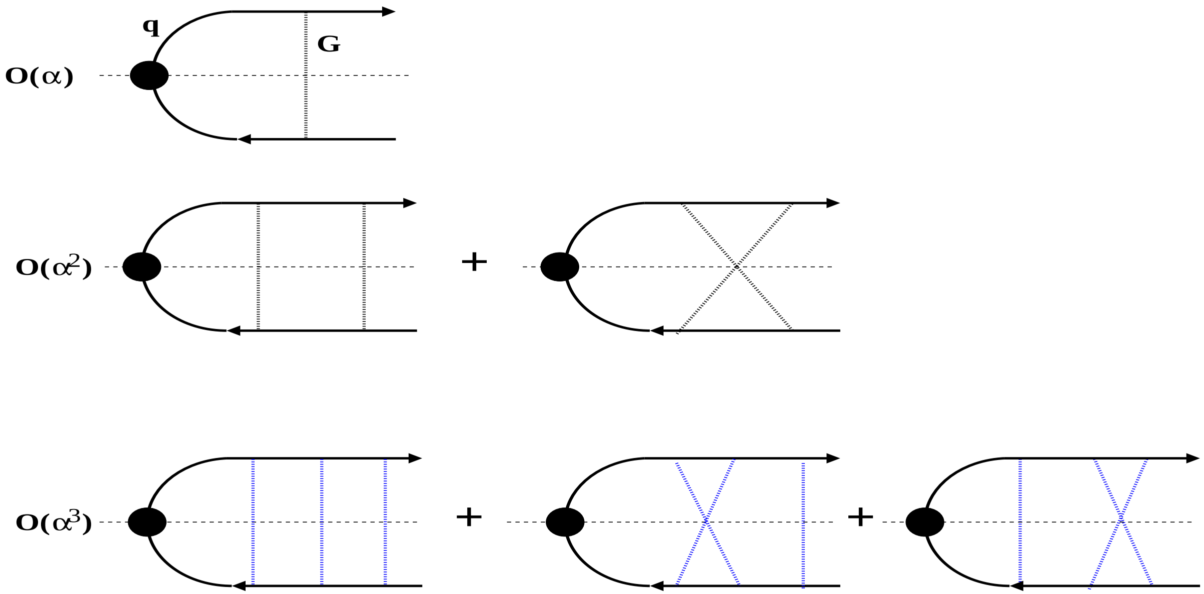
- Even more detailed analysis of many choices of $\Theta(Q^2)$, including rapitity ordering.
- Re-insertion of the NLO unintegrated kernel into LO MC model representing LO+NLO (DGLAP) evolution, for n=3,4 emissions and then for arbitrary n.
- Include C_A part of the gluonstrahlung in the game.
- Include singlet part in the game.
- Examine/reinvent “coefficient function” part (hard process) and include it in the MC model for DIS and DY.
- And more! New class of MCs for QCD beyond LO wide open!-))

Insertion of unintegrated NLO kernel into LO MC for n=3

Very schematically: MC with 1 NLO integr. kernel at n=1,2 emissions

$$e^{T(\alpha\mathcal{P}_1+\alpha^2\mathcal{P}_2)} = 1 + \alpha T\mathcal{P}_1 + \alpha^2\left(\frac{1}{2!}T^2\mathcal{P}_1^2 + T\mathcal{P}_2\right) + \alpha^3\left(\frac{1}{3!}T^3\mathcal{P}_1^3 + \frac{2}{2!}T^2\mathcal{P}_1\mathcal{P}_2\right) + \ldots$$

and MC with 1 UNintegrated NLO kernel at n=2,3 emissions



To be programmed/tested. Last nontrivial case is for n=4 emissions with 2 NLO kernels implanted in the MC...