QED calculations for Bhabha luminometer – – summary of LEP and lessons for the future

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Outline:

- **Main features of QED corrections for small angle Bhabha luminometer**
- **Brief review of the composition of the total "theoretical error" of the luminosity cross section at the end of LEP era.**
- **Lessons from LEP and predictions for the future Electron Linear Colliders (ELCs)**

Generic size of the photonic corrections, low angle Bhabha

$$\Delta \sim 2\frac{\alpha}{\pi} \int\limits_{m_e^2/s}^{|t|/s} \frac{d\theta^2}{\theta^2} \simeq 2\frac{\alpha}{\pi} \ln \frac{m_e^2}{|t|}$$

Possible extra increase by factor $\sim \ln(E_{\rm beam}/E_{\rm cut})$,

if the energy of the emitted photon is cut strongly, $E_\gamma < E_{\rm cut}$.

Here in Bhabha luminometry with double tag:

$$\ln(E_{\rm beam}/E_{\rm cut}) \simeq \ln(\vartheta_{\max}/\vartheta_{\min})$$

How many perturbative orders of QED do we need?

For the realistic description of Small Angle Bhabha with the double-tag calorimetric detection we need at least one photon collinear with the incoming electron (smearing acollinearity distribution) and one photon in the final state which is widening the calorimetric shower.

Each of them with $\sim 15\%$ probability, quite often in the same event. Hence two photons are mandatory! Implying second order $\mathcal{O}(\alpha^2)$.

In fact one need 3 or more soft photon for the realistic description of the distribution of acollinearity, angular distribution and shower profile.

Real emission we see in the detector. Invisible virtual emissions are equally important because they determine total normalization.

Exponentiation/resummation is mandatory!

Monte Carlo is mandatory!

Generic size of the photonic corrections, low angle Bhabha

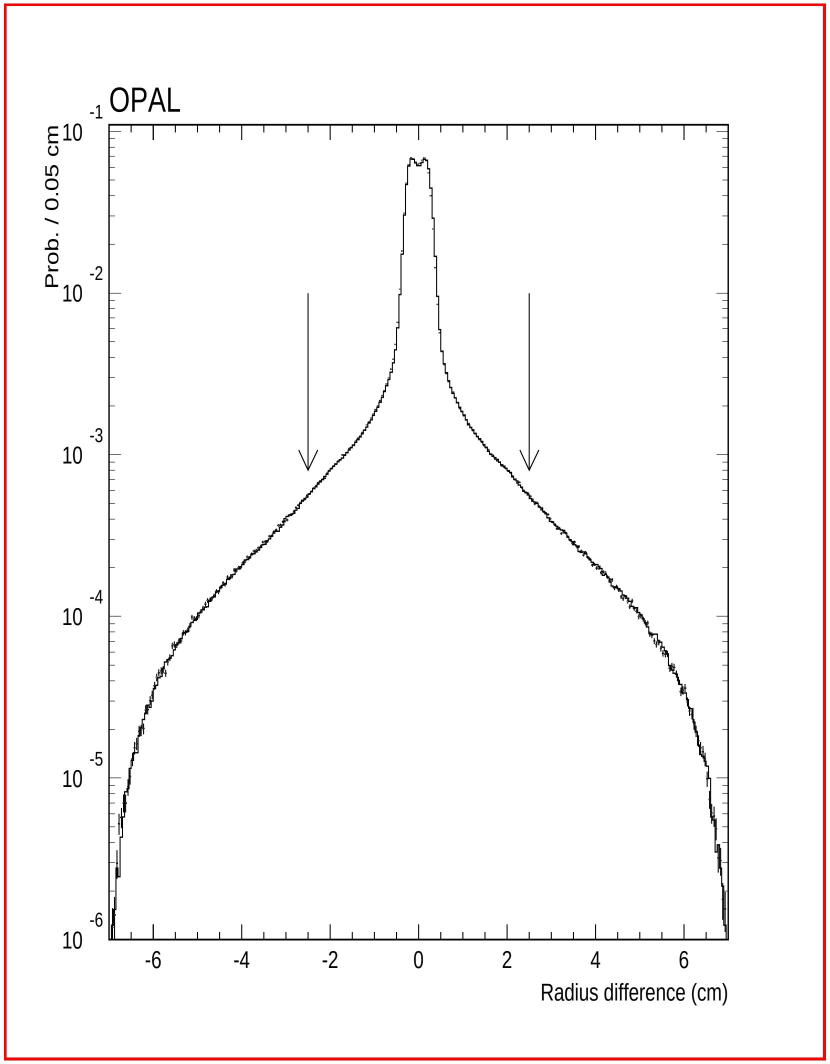
Canonical coefficients, $L = \ln(-t_{\min}/m_e^2) - 1$					
		θ_{min} = 30 mrad		θ_{min} = 60 mrad	
		LEP1	500GeV	LEP1	500GeV
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi}4L$	137×10^{-3}	168×10^{-3}	150×10^{-3}	181×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2}\frac{\alpha}{\pi}$	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}
$\mathcal{O}(\alpha^2L^2)$	$\frac{1}{2}\left(\frac{\alpha}{\pi}4L\right)^2$	9.4×10^{-3}	14.2×10^{-3}	11×10^{-3}	16×10^{-3}
$\mathcal{O}(\alpha^2L)$	$\frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}4L\right)$	0.31×10^{-3}	0.38×10^{-3}	0.35×10^{-3}	0.42×10^{-3}
$\mathcal{O}(\alpha^3L^3)$	$\frac{1}{3!}\left(\frac{\alpha}{\pi}4L\right)^3$	0.42×10^{-3}	0.78×10^{-3}	0.57×10^{-3}	1.01×10^{-3}

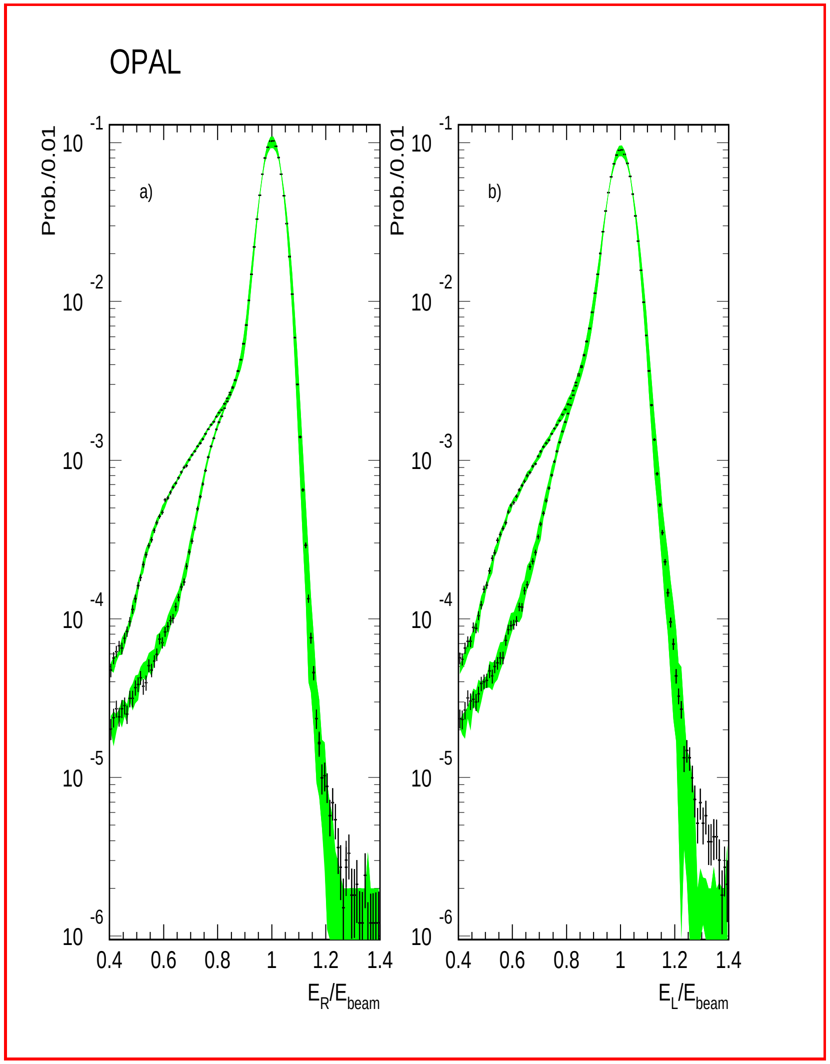
Already in 1992 MISSING photonic $\mathcal{O}(\alpha^2L)$ and $\mathcal{O}(\alpha^3L^3)$ anticipated ≤ 0.1%!

It took some time to prove it. Assuming similar angular range,

LEP$\longrightarrow$500GeV is expected to bring +20% to +60% in Photonic RCs.

Extraordinary agreement of OPAL data vs. BHLUMI MC, collinearity and energy distrs.





BHLUMI 4.04 Monte Carlo was used by all four LEP experiments. Not only controls luminosity normalization $d\sigma/\sigma = 0.06\%$, but also perfectly agrees with all experimental spectra, with NO "TUNING" to experimental data! (Only one bug in 1995.)

Legacy of BHLUMI, going beyond BHLUMI

At LEP luminosity measurement perfect agreement between BHLUMI MC and the data for energy of the final (dressed) electrons and of their collinearity was a cornerstone in reducing systematics experimental errors.
See for instance OPAL measurement.

At LC's situation will be dramatically different: the difference between distributions for experiment and MC will be exploited to measure beamstrahlung spectra! Experimentalist will have to have much more **"blind confidence"** in the lumi MC.

Another point: ANY new lumi MC will HAVE TO agree for the energy and angular spectra with BHLUMI at 91GeV, before it is seriously considered, because BHLUMI, effectively represents a "carbon copy" of the LEP1 experimental data (without beamstrahlung).

Any further improvement on lumi MC for ELC's beyond BHLUMI will require TWO independently developed MC's which agree perfectly for normalization and for all distributions.

BHLUMI: S. Jadach, E. Richter-Was, B.F.L. Ward, Z. Was; Comput.Phys.Commun. 70 (1992) 305

Past studies on RCs in luminometry and beamstrahlung at future ELCs

For TESLA there is a study by Klaus Moenig:

LC note LC-PHSM-2000-60-TESLA, December 2000,

which describes extraction of the beamstrahlung spectrum

using $d\sigma/d\theta_1 d\theta_2$ of the low angle Bhabha.

Similar study was done for CLIC energy 1.5TeV,

SNOWMASS-2001-E3015 by M. Battaglia, S. J. and D. Bardin.

The above studies are based on BHLUMI or BHWIDE Monte Carlo's supplemented with the "pre-generation" of the beam energy loss due to beamstrahlung.

NB. Does variation of the CMS energy destroy the MC algorithm of BHLUMI?

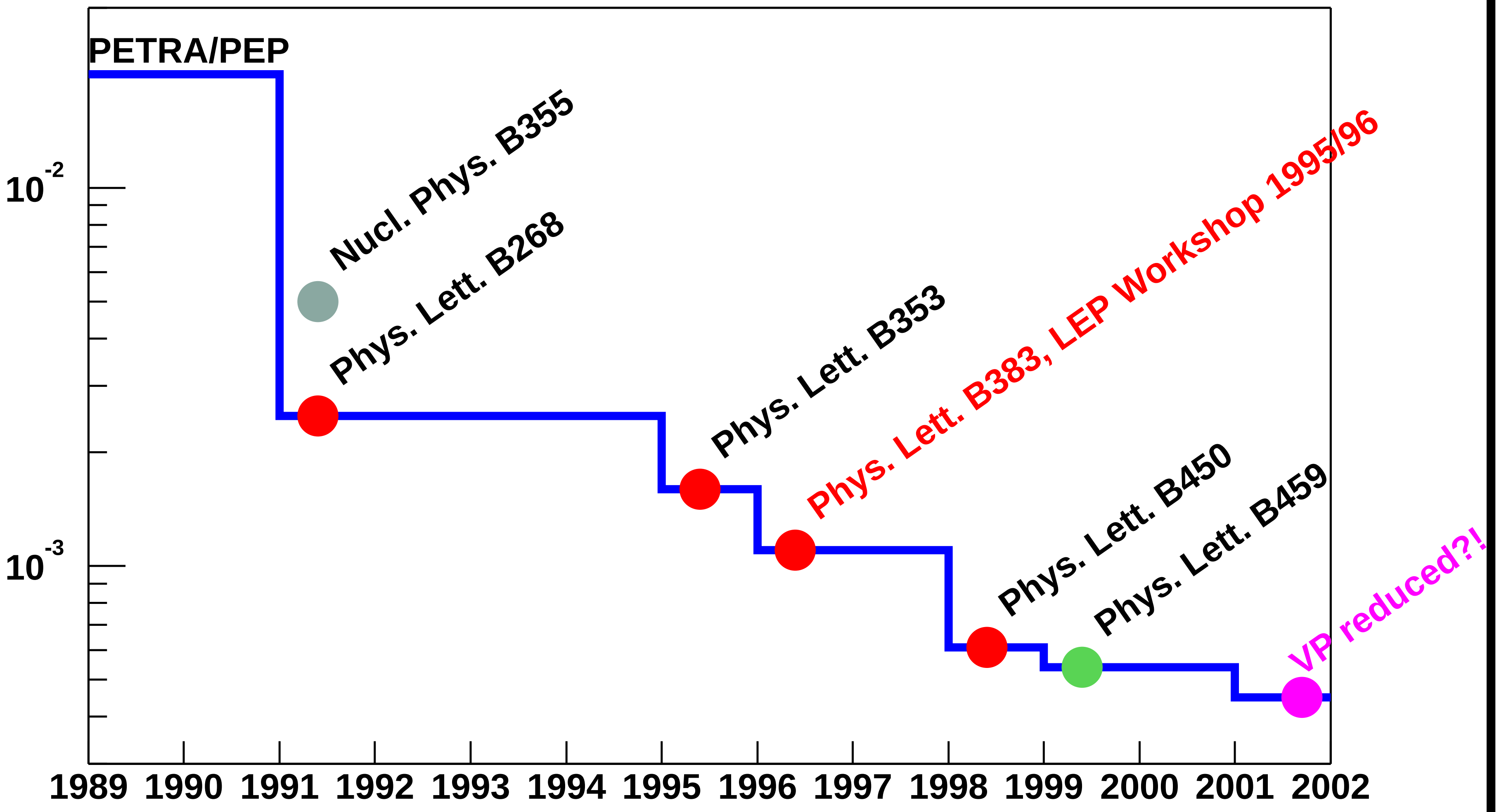
Probably not much or (with a little bit of luck not at all).

See also a few talks on http://jadach.home.cern.ch/jadach/

A Little bit of history:

Theoretical error of the luminosity at LEP era and its composition

Evolution of luminosity theoretical error at LEP1



● **LEP workshop 1995/96 (0.11%) was “The great consolidation”**

● **Latest published 1999 result: 0.054%** ● **Naive δVP=0.40→0.025% gives 0.044%**

References

Main TH precision improvements marked in red:

[1] W. Beenakker, F. A. Berends and S. C. van der Marck, Nucl. Phys. B **355** (1991) 281.
Photonic $\mathcal{O}(\alpha^2 L_e^2)$ and vacuum polarization (VP)

[2] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **253** (1991) 469.
Technical precision 0.02% establishe for the “baseline” $\mathcal{O}(\alpha^1)$ MC

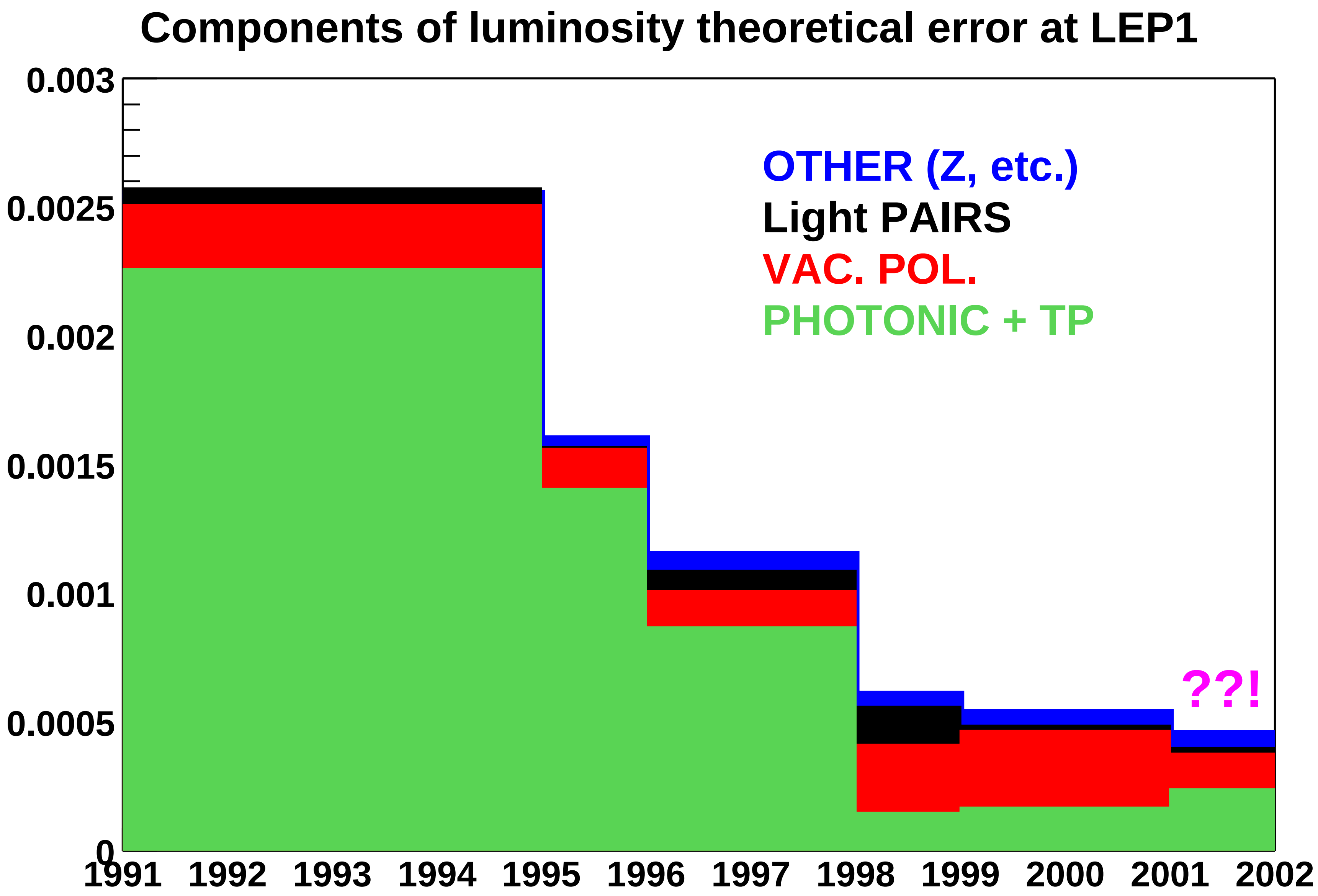
[3] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **260** (1991) 438.
First reliable estimate of the precision of $\mathcal{O}(\alpha^1)_{expon}$ multiphoton BHLUMI MC

[4] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **353** (1995) 362 [Erratum-ibid. B **384** (1996) 488].
Inclusion of $\mathcal{O}(\alpha^2 L_e^2)$ and $\mathcal{O}(\alpha^3 L_e^3)$, new estimate of $\mathcal{O}(\alpha^2 L)$

[5] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238
New estimate of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

[6] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262
New calculation of missing $\mathcal{O}(\alpha^2 L)$ in BHLUMI

[7] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649
New calculation of missing light real and virtual pairs



Components summed (usually) in the quadrature. Last entry under discussion!

Error budget at LEP Workshop 95/96, “The Great Consolidation” (in red)

	LEP1		LEP2
Type of correction/error	Ref.[1]	Ref. [2]	Ref.[2]
(a) Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	0.10%	0.20%
(a) Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.008%	0.015%	0.03%
(c) Vacuum polarization	0.05%	0.04%	0.10%
(d) Light pairs	0.04%	0.03%	0.05%
(e) Z-exchange	0.03%	0.015%	0.0%
Total	0.16%	0.11%	0.25%

[1] S. Jadach, E. Richter-Was, B. F. Ward and Z. Was, Phys. Lett. B **353** (1995) 362 [Erratum-ibid. B **384** (1996) 488].

[2] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238

My personal update of LEP1 theoretical error, Febr. 2003 (red/magenta)

Type of correction/error	Ref.[1]	Ref. [2]	Ref. [3]	My update
Technical precision	—	(0.030%)	(0.030%)	0.030%
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027%	0.027%	0.027%
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.015%	0.015%
Vacuum polarization	0.04%	0.04%	0.040%	0.025%
Light pairs	0.03%	0.03%	0.010%	0.010%
Z-exchange	0.015%	0.015%	0.015%	0.015%
Total	0.11%	0.061% (0.068)	0.054% (0.061)	0.053%

[1] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238

[2] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262

[3] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649

Examples of questions related to theoretical errors in Bhabha luminometry at future electron colliders

Conclusions of my previous talk at TESLA and CLIC WG meetings

Conclusions from numerical results:

- Pure QED photonic RCs and their errors only $\sim 30\%$ bigger than at LEP1
- EW corrections can be important. At 3TeV EW uncertainty $< 0.1\%$, provided state of art $\mathcal{O}(\alpha^1)$ EW are included!
- Error due to hadronic vacuum polarization $\sim 0.1\%$ seems to dominate
- Exponentiation for photonic QED RCs unavoidable!

QUESTION: Do we expect problems with theory error at the level of 0.1%

in the luminosity measurement using double-tagged Bhabha

within 25-100mrads, at 1-3 TeV?

ANSWER: Total error $\delta\sigma/\sigma < 0.1\%$ seems feasible, but...

will require serious work! What about GigaZ option?

The case of 0.5TeV collider is relatively easy, except of beamstrahlung.

Leading TH errors of σ^{tot}_{SABH} at Linear Colliders, 25-100mrad:

The **double-tag small angle Bhabha (SABH)** process is a leading candidate for the luminometer process at Linear Colliders.

Main theoretical uncertainties of SABH luminometer σ^{tot} at TESLA/NLC/CLIC, at scat. angles 25-100mrads:

- Hadronic vacuum polarization
- QED photonic corrections
- EW corrections to Z_t
- Light fermion pairs??

Main sources of TH error of σ^{tot}_{SABH} at ELCs, 25-100mrad:

Basic difference with LEP1:

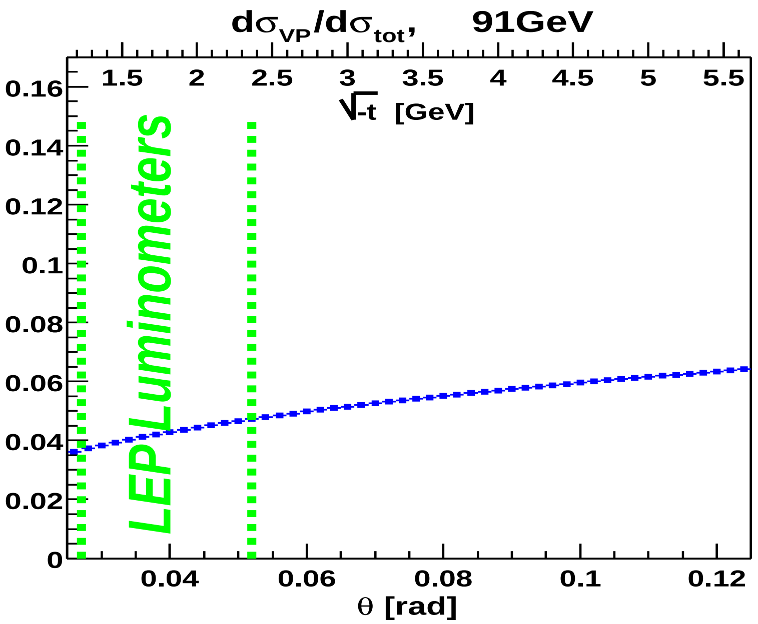
- The transfer instead of $\sqrt{|t|} \sim 2$GeV, for $\sqrt{s}$=91GeV and ϑ =45mrad, is at the same angle 10GeV at 500GeV, and 60GeV at 3TeV!
- Z (t-channel) increasingly (with $\sqrt{s}$) important!
- Hadronic vacuum polarization and its error grows strongly with $\sqrt{|t|}$, factor 2 grow from 100 to 500GeV and stabilizes > 500GeV.
- Photonic QED corrections $\sim \alpha \ln(|t|/m_e^2) \ln(\vartheta_{max}/\vartheta_{\min})$ grow mildly, and will increase by $\sim \ln(s/M_Z^2)/\ln(M_Z/m_e) \sim 15-30\%$.

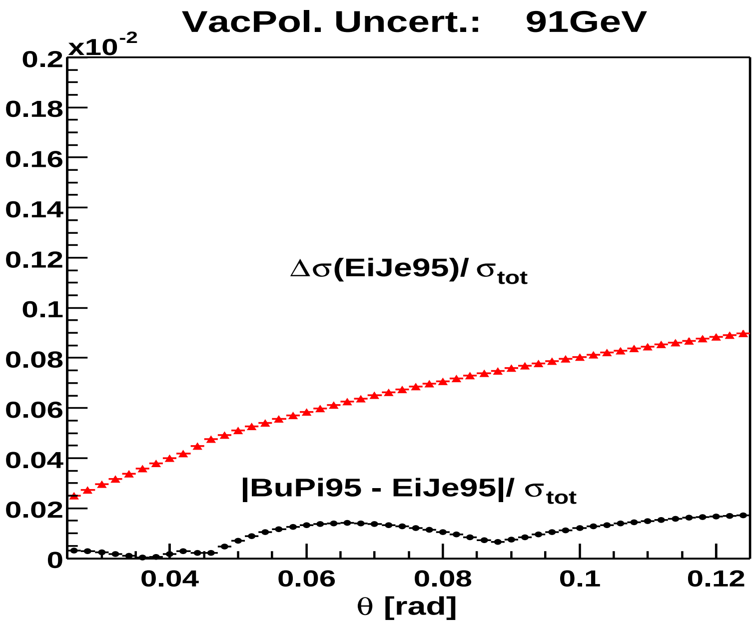
Last not least:

Due to beamstrahlung luminosity is a function $\mathcal{L}(z_1, z_2)$, not a number!

How big is Lumi uncertainty due to hadronic vacuum polarization?

Let us recall situation at LEP1:

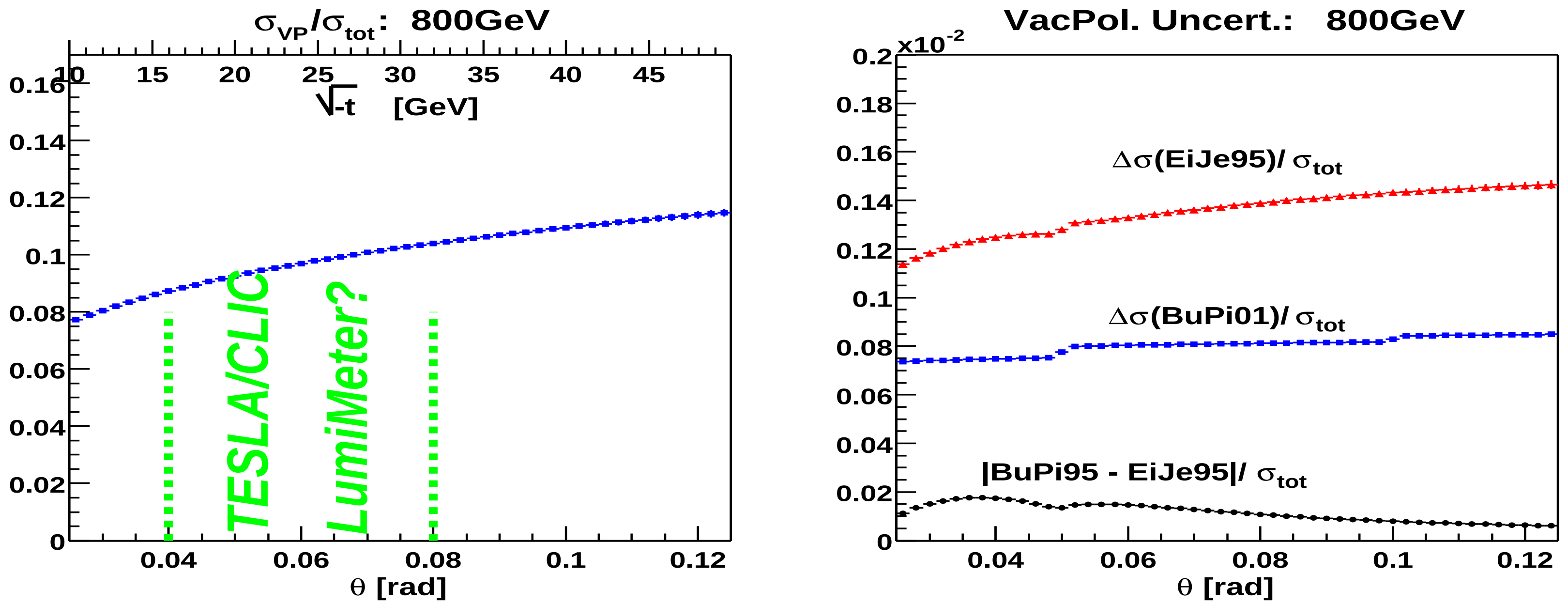




Next slide will show the case of 800GeV...

At 1-2GeV situation is almost the same.

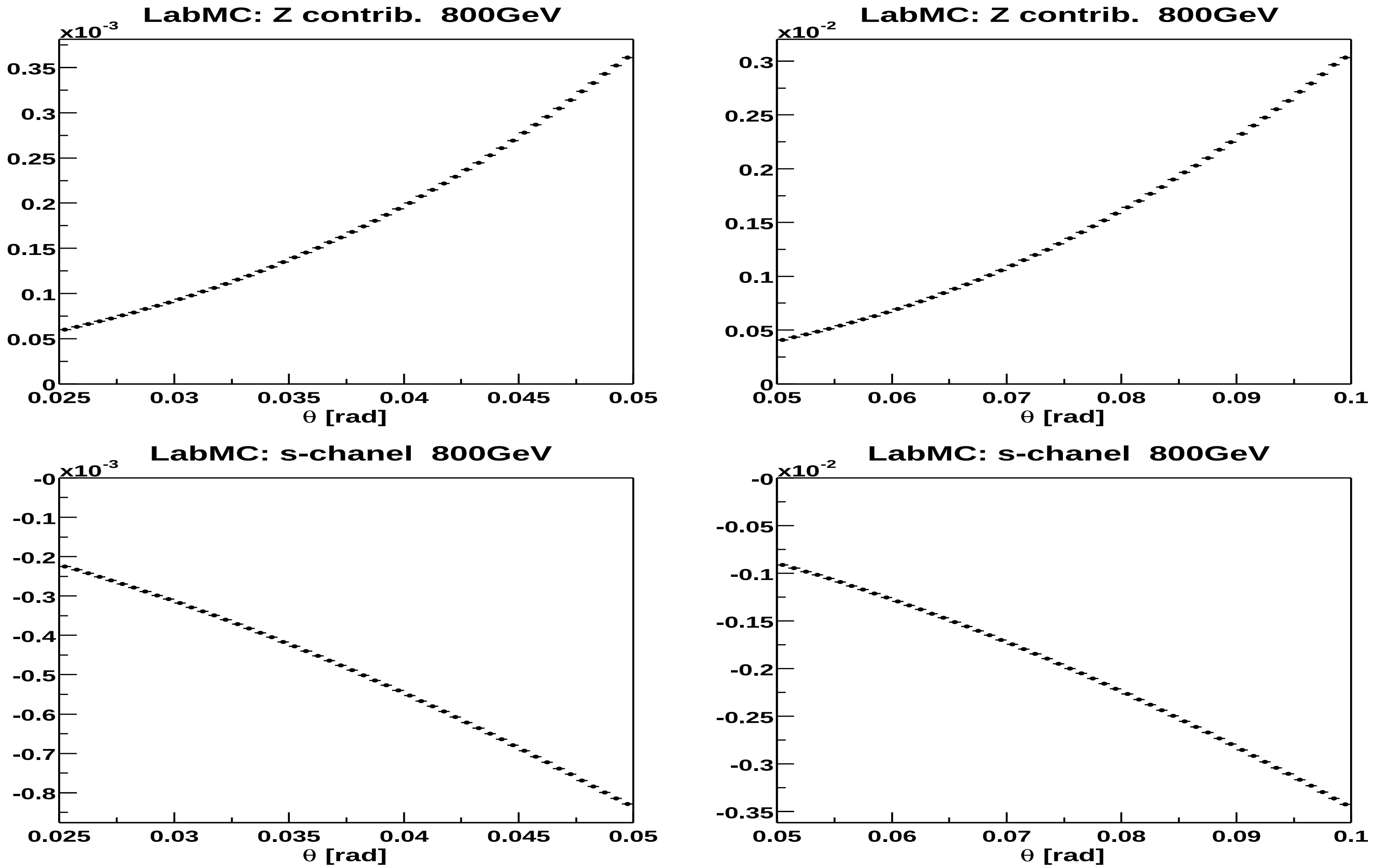
How big is Lumi uncertainty due to hadronic vacuum polarization?



- Plots from LabMC with updated Born&VacPol of BHWIDE.
- Not all latest improvements included (to be done).
- Also x-checked with DIZET 6.35 (using Eidelman& Jegerlehner 1995).

TH/Exper uncert. from Hadronic VP almost back to LEP1, thanks to recent works!

Z contribution and s-channel at 0.8TeV

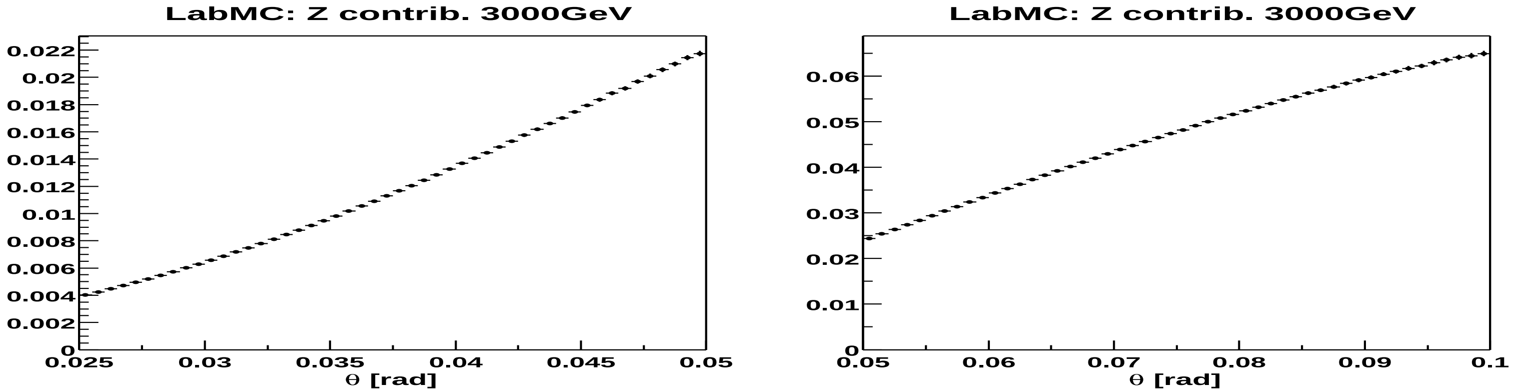


This result was x-checked with 2 Born-level calcul. using BHWIDE and DIZET.

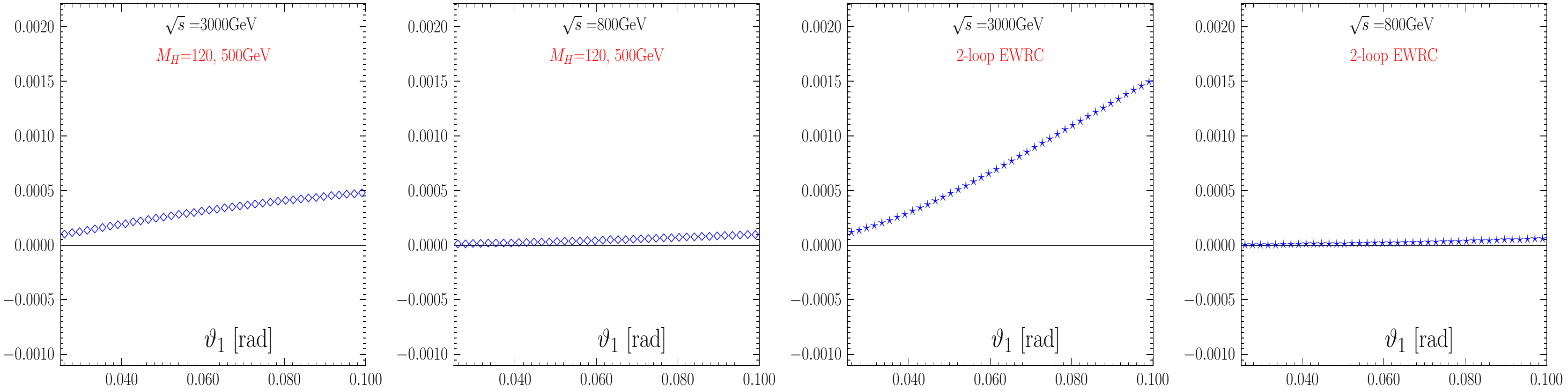
Polution due to s-channel and Z contributions are negligible $< 3 \times 10^{-3}$ at 800GeV.

Z contr. is sizeable beyond 1TeV, see next slide...

At 3TeV Z contribution (t-channel) is sizable, up to 6%.



How big is, therefore, uncertainty of due to EW corrections?



DIZET: Varied M_H =120⟶500GeV, M_t =165⟶185GeV and NPAR(2)=3⟶NPAR(2)=4 which manipulates non-leading 2-loop EW corrections $\mathcal{O}(G_F^2 M_t^2 M_Z^2)$, Degrassi et.al., keeping 2-loop EW corrections $\mathcal{O}(G_F^2 M_t^4)$.

At 3TeV (50-100mrad) we find TU of 0.09%, an estimate from NLL 2-loop EWRs.

Is exponentiation important?

If someone still thinks that

the so-called exact complete $\mathcal{O}(\alpha^2)$ calculation

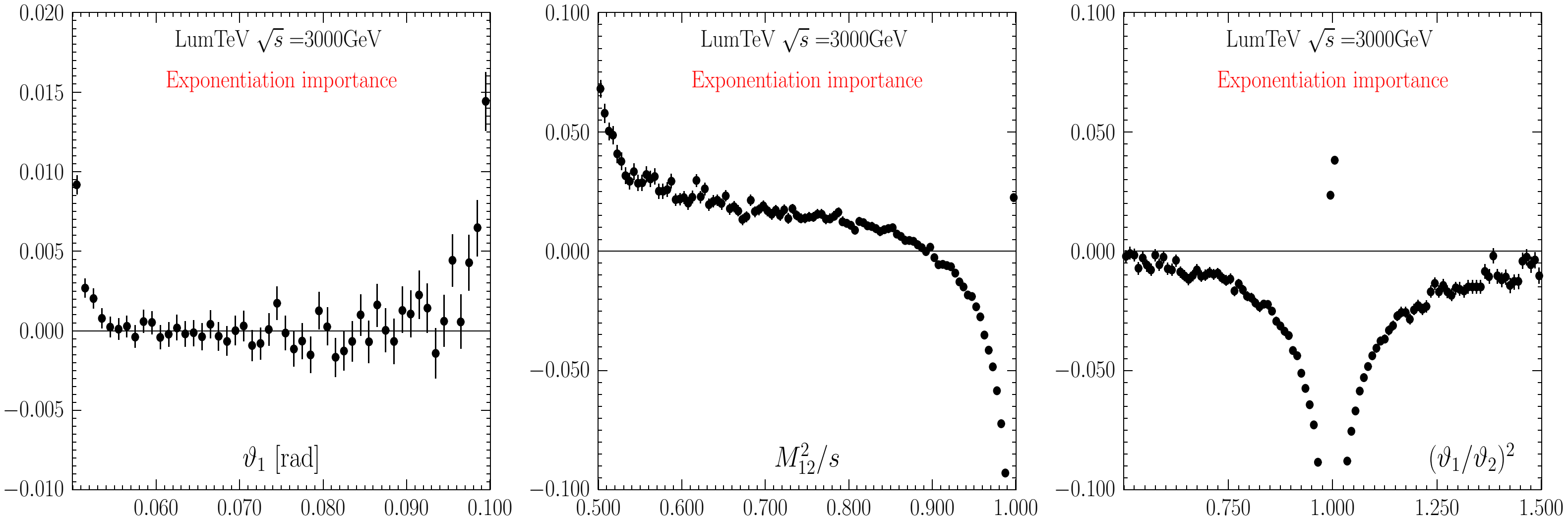
without exponentiation is good enough for the Small

Angle Bhabha precise prediction, then he should

consult the following slide...

Is exponentiation important for photonic QED r.c.'s?

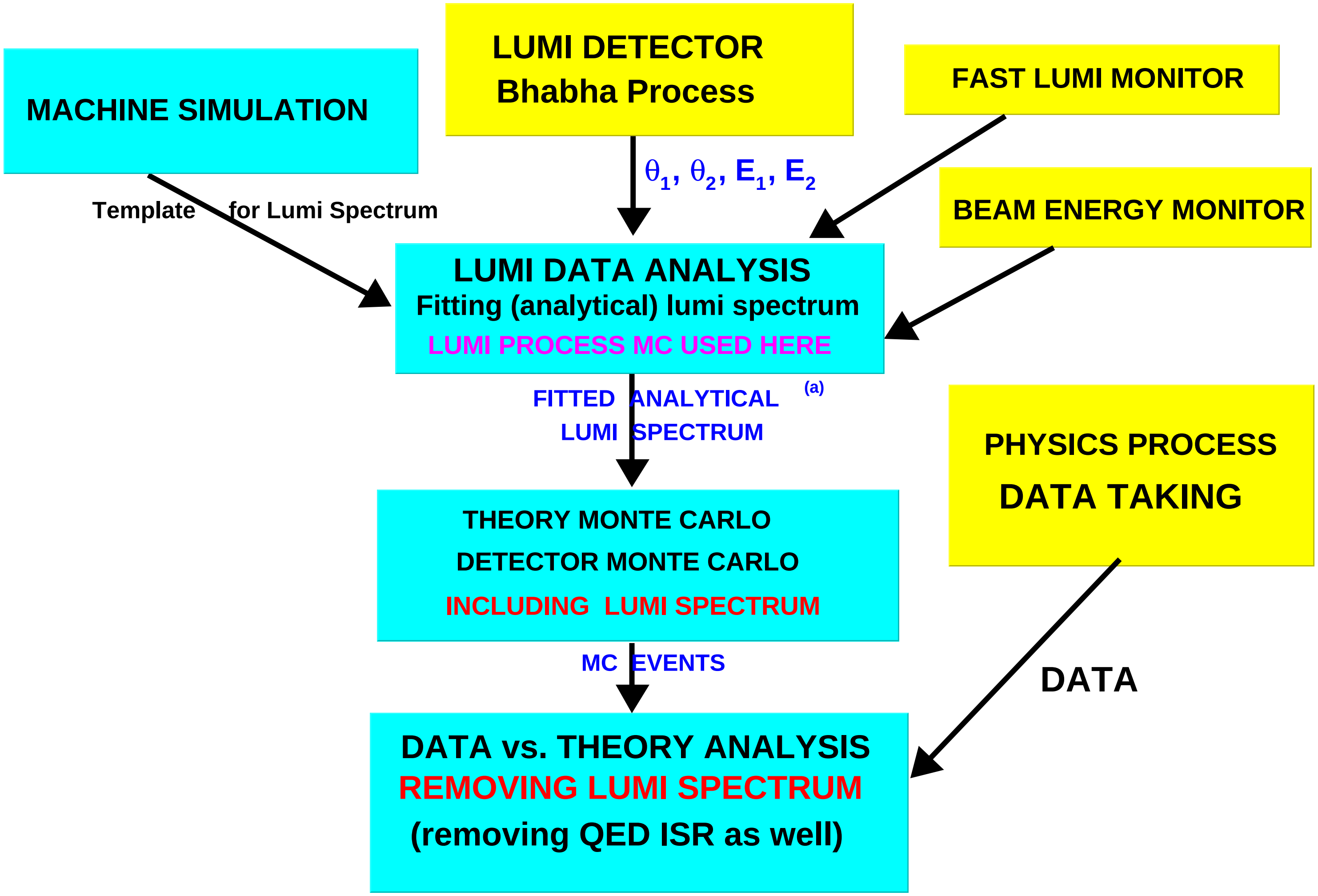
The difference $\mathcal{O}(\alpha^3)_{exp} - \mathcal{O}(\alpha^2)$ in LL approximation (ISR only) gives us hint how bad the calculation in $\mathcal{O}(\alpha^2)$ without exponentiation actually would be:



Conclusion: Exponentiation of photonic QED is absolute necessity!

Note that $M_{12}/s = z_1 z_2$ and $\vartheta_1/\vartheta_2 \simeq z_1/z_2$ are basic variables for determination of the luminosity distribution. Effects close to ϑ-edges are due to soft ISR photons.

Handling Lumi spectra in the future Real Experiment



MACHINE SIMULATION

Template for Lumi Spectrum

LUMI DETECTOR
Bhabha Process

$\theta_1, \theta_2, E_1, E_2$

FAST LUMI MONITOR

BEAM ENERGY MONITOR

LUMI DATA ANALYSIS
Fitting (analytical) lumi spectrum
LUMI PROCESS MC USED HERE

FITTED ANALYTICAL LUMI SPECTRUM (a)

THEORY MONTE CARLO
DETECTOR MONTE CARLO
INCLUDING LUMI SPECTRUM

MC EVENTS

PHYSICS PROCESS
DATA TAKING

DATA

DATA vs. THEORY ANALYSIS
REMOVING LUMI SPECTRUM
(removing QED ISR as well)

(a) The only viable VEHICLE for transferring the information about LUMI spectrum from the LUMI detector to physics MC event generator is a “parametric” representation $\mathcal{L}(a_1 ... a_n; z_1, z_2)$ fitted using MC.

Direct use of QED+beamstrahlung SF’s deduced in Lumi data analysis unfeasible, LL scale evolution $t \longrightarrow s$ and complications in controlling NLL’s.

Summary

- **Present theoretical error of low angle Bhabha at LEP $\simeq$0.05%-0.07% solid as ever. Room for an easy improvement exists (vacuum polarization).**
- **Radical improvement of the TH precision to $\leq 0.020\%$ (GigaZ?), i.e. below the best experimental error 0.034% of LEP seems feasible.**
- **This would require: reduction of the technical precision and of the photonic QED, including better Z-exchange for $\geq$800GeV (VP will get reduced another factor 2 in the meantime.)**
- **Somehow we have made a "full circle", and once again the technical precision and the photonic corrections are "the king".**
- **At future linear colliders all "alchemy" of the QED corrections to luminometry looks pretty much the same, except of their $\sim$30% increase and beamstrahlung**
- **The great problem of the extraction of the beamstrahlung spectra using Bhabha luminometer is still in the infancy. It will require much stricter standards for the whole MC evaluation of the QED RCs!**