Exclusive modelling of NLO DGLAP QCD Evolution in the NLO Parton Shower Monte Carlo

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in collaboration with

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More on http://jadach.web.cern.ch/

Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form; Mission statement:

- Three-decades old paradigm: hard process matrix element calculated to LO+NLO(+NNLO) level is combined with:
  1. either the collinear PDF at LO+NLO(+NNLO)
  2. or with the Monte Carlo parton shower, **but the MC PS is restricted to LO only! Forever?**
- Upgrading Monte Carlo parton shower to NLO level is regarded as **unfeasible in practice or in principle**, or both.
- For NLO non-singlet subset of diagrams we implemented in the Monte Carlo PS the NLO DGLAP evolution, in the exclusive/unintegrated form. Positive MC weights.
- Possibly, 1-st step to a new class of powerful techniques of combining resummed and finite order pQCD calculations in a form of MC event generators, for LHC and beyond.

DGLAP Collinear QCD ISR Evolution in the Monte Carlo

1970 1980 1990 2000 2010

LO

Moments OPE: (74) QCD: Georgi+Politzer

Diagramatic: (72) QED: Gribov+Lipatov; (77) Altarelli+Parisi

Monte Carlo: 10 years → (85) Sjostrand; (88) Marchesini,Webber

NLO

WE ARE HERE!!!

Moments OPE: (78) Floratos+Ross+Sachrajda

Diagramatic: (81) Curci+Furmanski+Petronzio

Monte Carlo: (92) Kato et.al.; 27 years later → (08) Jadach Skrzypek

NNLO

Moments: (03) Moch+Verm.+Vogt

Diagramatic: (03) Moch+Verm.+Vogt

Monte Carlo: (15) ???

Why 20 years time lag?

Many reasons:

1. Lack of motivation – poor exp. data from hadron colliders
2. QCD Parton shower Monte Carlo (PSMC) main objective was (still is?) hadronization
   – until recently not involved in pQCD @ hard process
3. Conceptual barriers: Factorization theorems (EGMPR, CFP, CSS, Bodwin,...) not suited for MC beyond LO:
   - non-conservation of 4-momenta
   - over-subtractions $\rightarrow$ huge cancellations
   - non-positive distributions
   - real emissions irreversibly integrated over
4. CPU time: in 1985 No. of MC events <50k, $\sim$ 3% precision; now 2010, $2 \cdot 10^{10}$ events and $\sim$ 0.01% errors are routine.

Our expection: After completing NLO and NNLO calculations for hard processes NLO PSMC will be the main front of pQCD activity?!

Possible profits/gains from NLO PSMC

- Complete set of “unitegrated soft counterterms” for combining hard process ME at NNLO with NLO PS MC
- Natural extensions towards BFKL/CCFM at low x
- Better modelling of low scale phenomena, $Q < 10GeV$, quark thresholds, primordial k^T, underlying event, etc.
- Porting information on parton distributions from DIS (HERA) to W/Z/DY (LHC) in the MC itself, instead in form of collinear PDFs (universality must be preserved)
- and more...

MC modelling of NLO DGLAP is not the aim in itself – it will be a starting platform for many developments in many directions.

The aim of the present exercise (KRKMC project)

Constructing NLO Parton Shower Monte Carlo for QCD Initial State Radiation for one initial parton:

- based on the collinear factorization (EGMPR, CFP, CSS, Bodwin...) as rigorously as we can,
- CFP=Curci-Furmanski-Petronzio scheme as a main reference/guide (axial gauge, $\overline{MS}$ dim. regulariz.),
- implementing *exactly* NLO DGLAP evolution,
- and fully unintegrated exclusive PDFs (ePDFs),
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels

Proof of the concept for non-sing. NLO DGLAP.

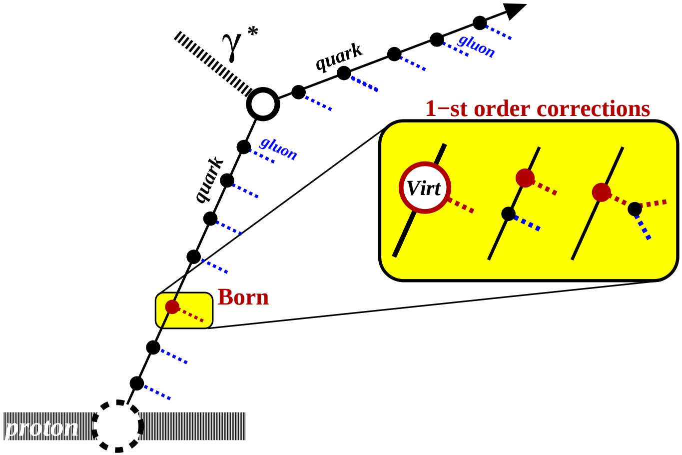
More details on the project are available here:

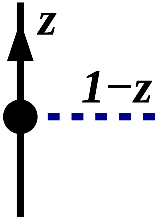
- Epiphany 2009 Proceedings,
  article http://arxiv.org/abs/0905.1399
  slides http://home.cern.ch/jadach/public/epip09.pdf
  More on re-inserting NLO corrections into LO MC ($\sim C_F^2$)...
- RADCOR 2009 Proceedings,
  article http://arxiv.org/abs/1002.0010
  slides http://home.cern.ch/jadach/public/RADCOR09.pdf
  More on the “factorization scheme” in our NLO PSMC versus CFP/EGMPR...

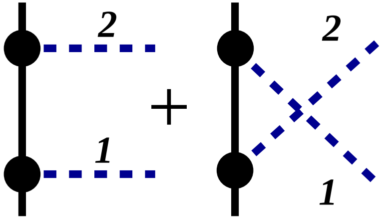
Leading Order (LO) ladder vertex is our “Born”

Emission of gluons out of quark

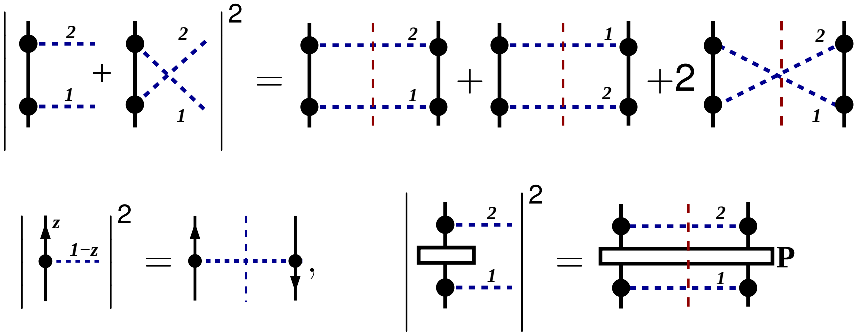
1-st step: Implementing in the Monte Carlo complete NLO DGLAP in the initial state ladder, using unintegrated Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge).



Virtual : $\left(1 + \Delta^{(1)}_{ISR}(z)\right)$ 

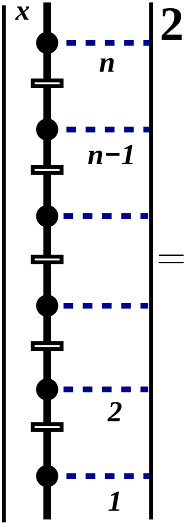
Real : 

NOTATION: squared MEs = cut-diagrams, C_F^2 only



LO ladder = parton shower MC

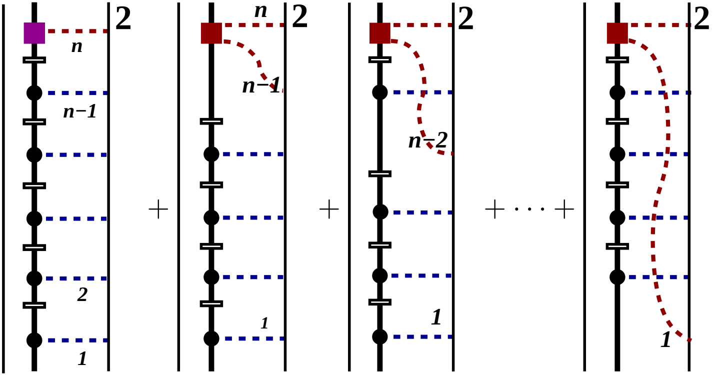
$$\sum_{n=0}^{\infty}$$



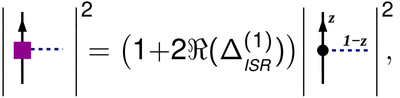
$$= e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0}\, \theta_{Q>a_i>a_{i-1}}\, \rho_{1B}^{(0)}(k_i)\, \delta_{x=\prod z_i}$$

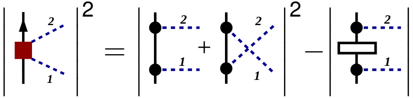
$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}.$$

LO with NLO-corrected kernel at the end of the ladder



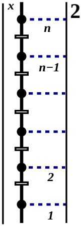
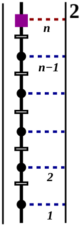
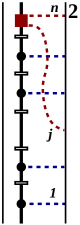
Virt. multiplicative Undoing LO simplificat. Sum over trailing LO spectators, essential (BE, YFS61)



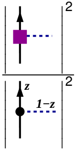
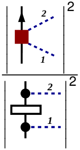
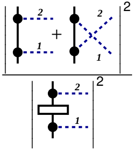


LO with NLO-corrected end-ladder kernel, $\sim C_F^2$

MORE DETAILS:

$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}}$  $+\, e^{-S_{ISR}}$  $+\, e^{-S_{ISR}} \sum_{j=1}^{n-1}$  $= e^{-S_{ISR}} \Big\{ \delta_{x=1} +$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Big\},$$

where $d\eta_i = \frac{d^3 k_i}{k_i^0}$, $\quad \beta_0^{(1)} =$ , $\quad W(k_2,k_1) =$  $=$  $-\,1.$

Mapping $k_i \to \tilde{k}_i$ instrumental. $\quad S_{ISR} =$ double-log Sudakov, $\quad W$ is non-singular!

Algebraic crosscheck

Analytical integration of NLO part $\sum_j W(\tilde{k}_n, \tilde{k}_j)$ can be done leading to:

$$\sum_{n=1}^{\infty} \int du \int\limits_{Q>a_n>a_{n-1}} \frac{da_n}{a_n} \mathcal{P}^{(1)}_{qq}(u) \left(\prod_{i=1}^{n-1} \int\limits_{a_{i+1}>a_i>a_{i-1}} \frac{da_i}{a_i} \mathcal{P}^{(0)}_{qq}(z_i) \right) \delta_{x=u\prod_{j=1}^{n-1} z_j}.$$

where we recover precisely NLO part (including virtuals) of standard DGLAP kernel $\mathcal{P}^{(1)}_{qq}(u)$ defined according to:

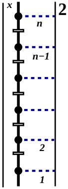
$$\mathcal{P}^{(1)}_{qq}(u) \ln\frac{Q}{q_0} = \int\limits_{Q>a_n>a_0} d^3\eta_n\ \rho^{(1)}_{1B}(k_n)\ \beta^{(1)}_0(z_n)\delta_{u=z_n} + \int\limits_{Q>a_n>a_0} d^3\eta_n \int\limits_{a_n>a_{n'}>0} d^3\eta_{n'}\ \beta^{(1)}_1(\tilde{k}_n, \tilde{k}_{n'})\ \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced.

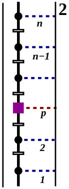
NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p:

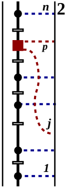
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p=1}^{n}$$



$$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$



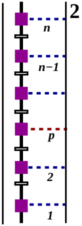
$$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

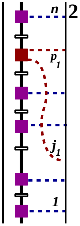
Next step is to add more “NLO insertions”,
for instance 2 at positions p_1 and p_2 and sum up over them...
then 3 insertions at p_1, p_2, p_3 and so on
– LO+NLO kernels built up all over along the ladder!

NLO-corrected kernels all over the ladder, $\sim C_F^2$

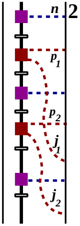
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1}$$



$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1\\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1\\ j_2\neq p_1, j_2}}^{p_2-1}$$



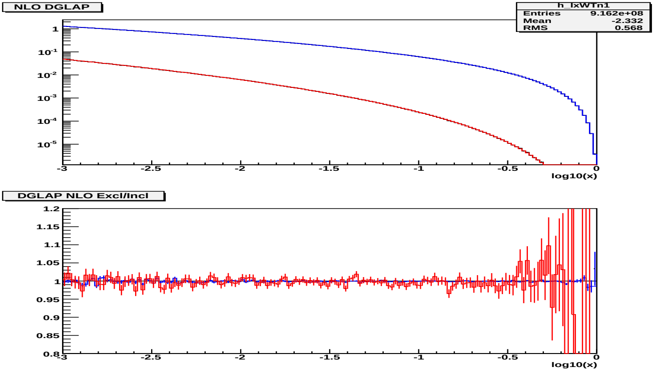
$$\Bigg\}$$

$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i\ \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \Bigg) \Bigg[1 + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) +$$

$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1\\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1\\ j_2\neq p_1, j_2}}^{p_2-1} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \dots \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.

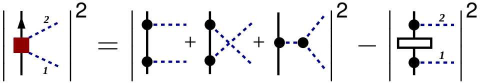
Numerical test of ISR pure C_F^2 NLO MC



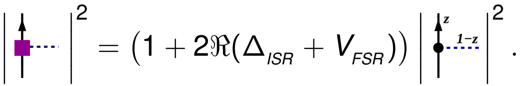
Numerical results for $D(x, Q)$ from inclusive and exclusive **two** Monte Carlos.
Blue curve is single NLO insertion, red curve is double insertion component.
LO+NLO is off scale. Evolution 10GeV$\rightarrow$1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

THE PROBLEM WITH GLUON PAIR COMPONENT OF the NLO KERNEL, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:



and $-S_{FSR}$ in the virtual correction:

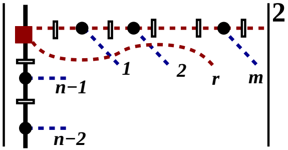


SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

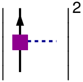
NLO FSR corr. at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR corr. at the end of the ladder:

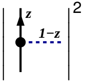
$$e^{-S_{ISR}-S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$



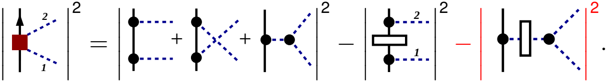
where Sudakov S_{FSR} is subtracted in the virtual part:



$$= \left(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\right)$$



and FSR counterterm is subtracted in the 2-real-gluon part:

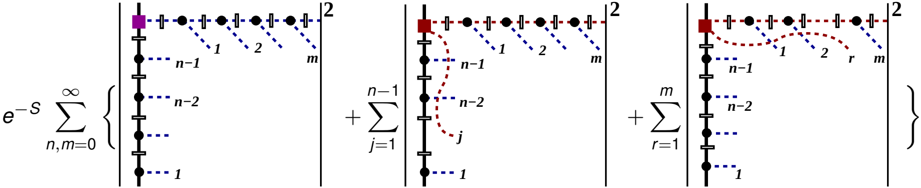


The miracle: both are free of any collinear or soft divergence!!!

Important point in this talk, next slide:

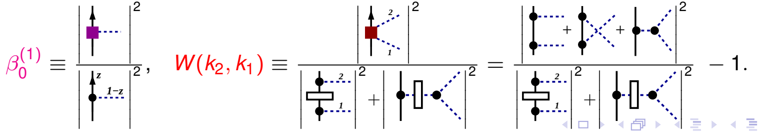
ISR+FSR NLO scheme, NLO corr. at end of the ladder

$\bar{D}^{[1]}_{NS}(x,Q) =$



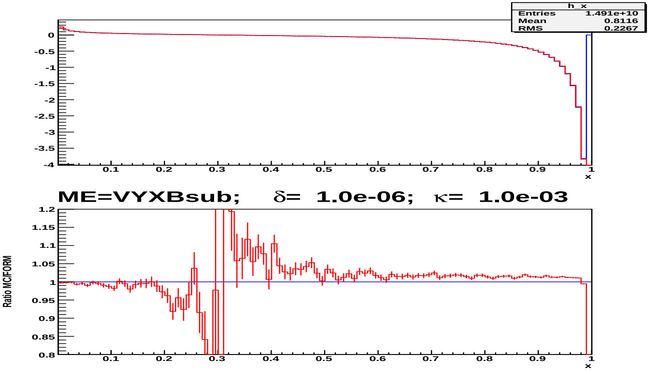
$$= e^{-S_{ISR}}\Big\{\delta_{x=1}+\sum_{n=1}^{\infty}\Big(\prod_{i=1}^{n}\int_{Q>a_i>a_{i-1}} d^3\eta_i\;\rho^{(1)}_{1B}(k_i)\Big)e^{-S_{FSR}}\sum_{m=0}^{\infty}\Big(\prod_{j=1}^{m}\int_{Q>a_{nj}>a_{n(l-1)}} d^3\eta'_j\;\rho^{(1)}_{1V}(k'_j)\Big)$$

$$\times\Big[\beta_0^{(1)}(z_n)+\sum_{j=1}^{n-1}W(\tilde{k}_n,\tilde{k}_j)+\sum_{r=1}^{m}W(\tilde{k}_n,\tilde{k}'_r)\Big]\delta_{x=\prod_{j=1}^{n}x_j}\Big\}$$



3-digit precision numerical test of FSR methodology

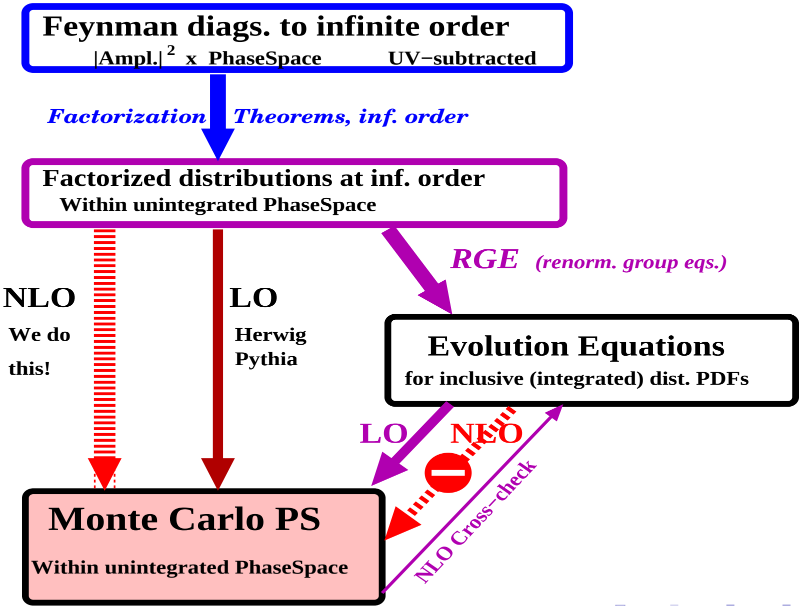
Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:



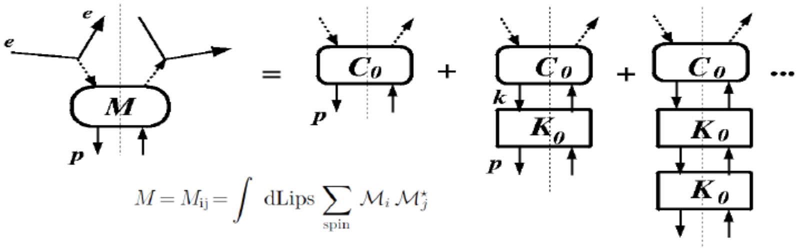
because in this case analytical integration is feasible.
MC agrees precisely with the analytical result.

A few comments on the theory of the collinear factorization and the Monte Carlo

QCD factorization versus Monte Carlo - An overview



"Raw" factorization of the IR collinear singularities



$$M = M_{ij} = \int \mathrm{dLips} \sum_{\mathrm{spin}} \mathcal{M}_i \mathcal{M}_j^*$$

- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0(1 + K_0 + K_0^2 + \cdots) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

EGMPR scheme customized to $\overline{MS}$ by Furmanski and Petronzio (80):

$$F = C_0 \cdot \frac{1}{1-K_0} = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right),$$

$$= \left\{ C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0} \right\} \otimes \left\{ \frac{1}{1-\left(\mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)} \right\}_{\otimes},$$

$$\Gamma\left(\alpha, \frac{1}{\epsilon}\right) \equiv \left(\frac{1}{1-K}\right)_{\otimes} = 1 + K + K \otimes K + K \otimes K \otimes K + ...,$$

$$K = \mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}, \quad C = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}.$$

Ladder part Γ corresponds to MC parton shower
C is the hard process part
$\mathbb{P}$ is the projection operator: $\mathbb{P} = P_{spin}\, P_{kin}\, PP$

For MC we use right now brute force interpretation of collinear ε-poles:

$$\frac{1}{\varepsilon} = \int_0^{\mu_F} \frac{dk^T}{k^T} \left(\frac{k^T}{\mu_F}\right)^{\varepsilon}.$$

CFP (1980) factorization scheme

$$F = C_0 \cdot \frac{1}{1-K_0} = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0} \otimes \Gamma, \quad \Gamma = \frac{1}{1-\left(\mathbb{P}K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)},$$

introduces enormous oversubtractions/cancellations. At LO we have:

$$\Gamma \simeq \frac{1}{1-\left(1-e^{-\frac{1}{\varepsilon}}\right)} = 1 + \left(1-e^{-\frac{1}{\varepsilon}}\right) + \left(1-e^{-\frac{1}{\varepsilon}}\right)^2 + \ldots$$

while from RGE and explicit LO calculation give us directly

$$\Gamma = e^{+\frac{1}{\varepsilon}} = 1 + \frac{1}{\varepsilon} + \frac{1}{2!}\frac{1}{\varepsilon^2} + \ldots$$

We want this exponent directly from the Feynman diagrams!!!

$$\boxed{F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^s K_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu)}$$

Time ordered exponential:

$$\exp_{TO}\left(\mathbb{P}'_\mu\{A\}\right)(\mu) = 1 + \mathbb{P}'_\mu\{A\} + \mathbb{P}'_\mu\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\} + \mathbb{P}'_\mu\{{}^{s_3}A\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\} + \dots$$

NOTATION: For $A = \int dLips(k_1, k_2, ..., k_n)\, f(k_1, ..., k_n)$,
where k_i are on-shell cut lines (real emitted partons)
the notation $\{{}^{s_3}A\}$ defines $s_3 = a(a_1, ..., a_n) = \max(a_1, ..., a_n)$.

Hence, term like

$$\mathbb{P}'_\mu\{{}^{s_3}A\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}A\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}A\}$$

has its entire integrand multiplied by $\theta_{\mu > s_3 > s_2 > s_1}$,
where μ is constant and s_i are integration variables dependent.
The key point is definition of the $\mathbb{P}'_\mu$ proj. operator.

Evolution and kernels...

In our factoriz. formula $F(Q) = C(Q,\mu)\cdot D(\mu)$, hard process part is $C(Q,\mu) = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0]$. and exclusive PDF (ePDF) is the integrand in:

$$D(\mu) = \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu) = \exp_{TO}(K).$$

LO and NLO truncations of the exclusive evolution kernel K_μ are:

$$K^{LO}_\mu = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0\right\}, \quad \text{taken at } \mathcal{O}(\alpha^1),$$
$$K^{NLO}_\mu = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0 + K_0\cdot(1-\overleftarrow{\mathbb{P}}')\cdot K_0\right\}, \text{ truncated at } \mathcal{O}(\alpha^2).$$

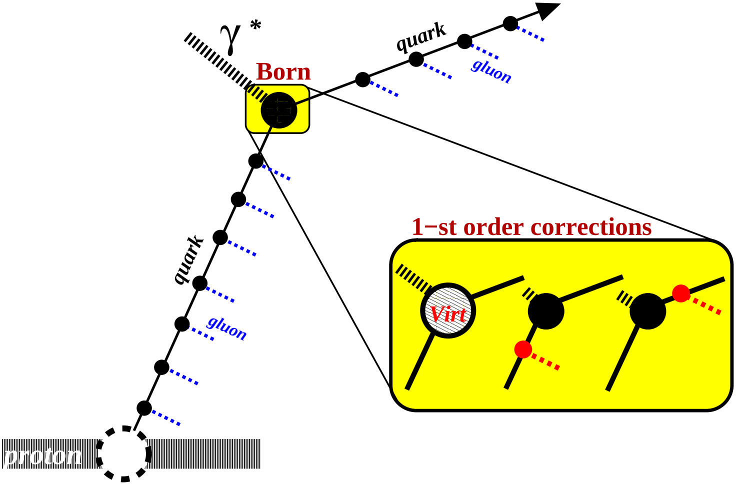
The x-dependent $D(\mu,x)$ obeys ordinary evolution equation:

$$\boxed{\partial_\mu D(\mu,x) = \mathcal{P}\otimes D(\mu)(x)}$$

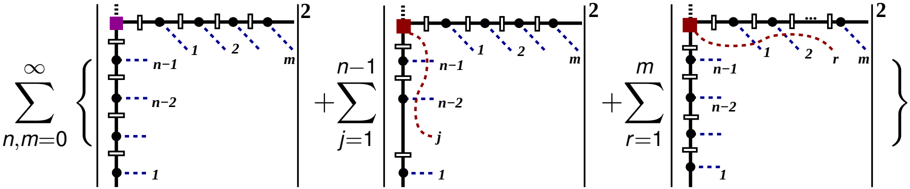
with the traditional inclusive DGLAP kernel being

$$\mathcal{P}(x) = \int d\mathrm{Lips}\,\delta\left(x = \frac{\sum k_i^+}{E_0}\right)\delta\left(1-\frac{s}{\mu}\right)\overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0\cdot\overleftarrow{\mathbb{R}}_s[K_0]\right\}.$$

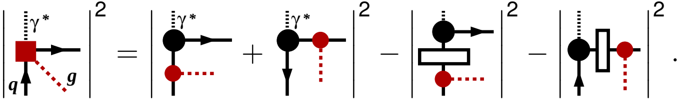
What next? NLO corrections to HARD process M.E.



General scheme of NLO corrections to HARD process

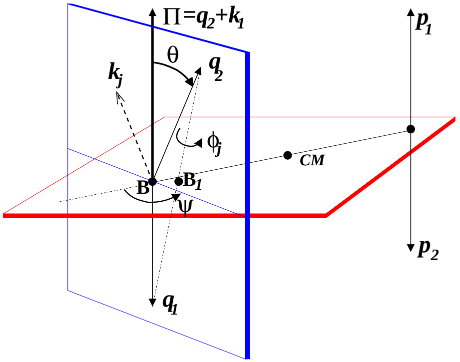


where the following NLO real/virtual corrections/distributions



are free of any double or single collinear/soft singularities.
More details on the following pages...

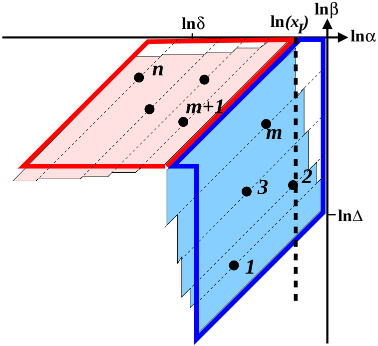
$$e(p_1) + q(q_1) \to e(p_2) + q(q_2) + g(k_1) + g(k_2) + ... + g(k_n).$$



The standard Sudakov variables are:

$$\alpha_i = \frac{k_i q_2}{q_1 q_2}, \quad \beta_i = \frac{k_i q_1}{q_1 q_2}, \quad \bar{\alpha}_i = \frac{\alpha_i}{1 + \sum_j \beta_j}, \quad \bar{\beta}_i = \frac{\beta_i}{1 + \sum_j \beta_j},$$

$$0 < \sum_j \bar{\alpha}_j \leq 1 - \frac{t}{s}, \quad 0 < \sum_j \bar{\beta}_j \leq 1$$



$$d\sigma_n^{LO} = Q_q^2\alpha_{QED}^2 \frac{dtd\varphi}{t^2}\ \frac{d\psi}{2\pi}\ \delta_y\Big(\sum_j \vec{k}_j\Big)\frac{1}{n!}\left(\prod_{i=1}^{n}\frac{C_F\alpha_S}{\pi}\ \frac{d\bar\alpha_i d\bar\beta_i}{\bar\alpha_i\bar\beta_i}\ \frac{d\phi_i}{2\pi}\bar{P}(z_i^k)\theta_{a_i>a_{i-1}}\right) e^{-S}\ W_0(\frac{y}{x_I})\theta_{\sum\bar\alpha_i<1}\theta_{\sum\bar\beta_i<1}$$

where $W_0(y) = 1+(1-y)^2$, $y = |t|/s$ and variables $z^k i = x_i/x_{i-1}$ are defined using $x_i = \sum_i^j \bar\alpha_i$ for ISR and using $x_i = \sum_i^j \bar\beta_i$ for FSR parts. The sudakov S is redefined as shade area blue for ISR, pink for FSR.

$$
d\sigma_n^{NLO} = Q_q^2 \alpha_{QED}^2 \frac{dt}{t^2}\, d\varphi\, \frac{d\psi}{2\pi}\, \delta_y\Big(\sum_j \vec{k}_j\Big)\, \frac{1}{n!} \left(\prod_{i=1}^{n} \frac{C_F \alpha_S}{\pi} \frac{d\bar{\alpha}_i d\bar{\beta}_i}{\bar{\alpha}_i \bar{\beta}_i}\, \frac{d\phi_i}{2\pi} \bar{P}(z_i^k) \theta_{a_i > a_{i-1}} \right)
$$
$$
\times\, \theta_{\sum \bar{\alpha}_i < 1} \theta_{\sum_{i\in F} \bar{\beta}_i < 1}\; e^{-S}\; W_0(y/x_I)(1+\Delta_{S+V})\;\; w_{MC}^{\Delta NLO},
$$

where

$$
w_{MC}^{\Delta NLO} = 1 + \sum_{i\in I} \frac{\bar{\beta}_I(\bar{\alpha}'_i, \bar{\beta}'_i)}{W_0(y/x_I))\bar{P}(1-\bar{\alpha}_i)} + \sum_{i\in F} \frac{\bar{\beta}_F(\bar{\alpha}''_i, \bar{\beta}''_i)}{W_0(y/x_I)\bar{P}(1-\bar{\beta}_i)},
$$

$\bar{\beta}_{I,F}$ are ISR, FSR-subtracted NLO 1-real gluon distributions, see next slide, Δ_{S+V} collects NLO virtual and soft corrs. We will recover at the inclusive level the structure function with exact NLO hard process matrix element (coeff. func.):

$$
\frac{d\sigma_{AP}^{NLO}}{dt dx_{Bj}} = \frac{2\pi Q_q^2 \alpha_{QED}^2}{t^2} \int_0^1 \int_0^1 dxdz\; \delta_{x_{bj}=xz}\; D_{\mathcal{I}}^{LO}(t,x)\; \hat{C}_{AP}^{NLO}(z, y/x_{Bj}).
$$

Special definition of $\bar{\alpha}'_i, \bar{\beta}'_i$ and $\bar{\alpha}''_i, \bar{\beta}''_i$ is another key point.

DIS with complete NLO exclusive at hard process

Raw material for NLO distributions

The NLO-complete **unsubtracted** distribution is

$$d\sigma_1^{NLO} = Q_q^2 \alpha_{QED}^2 dt d\varphi \; \frac{1}{t^2} \; \frac{d\psi}{2\pi} \; \delta_y\left(\vec{k}_1\right) \; \frac{C_F \alpha_s}{\pi} \frac{d\bar{\alpha}_1 d\bar{\beta}_1}{\bar{\alpha}_1 \bar{\beta}_1} \; \frac{d\phi_1}{2\pi} \left\{ W(\bar{\alpha}_1, \bar{\beta}_1, y/(1-\bar{\alpha}_1)) \right\},$$

$$W(\bar{\alpha}_1, \bar{\beta}_1, y/(1-\bar{\alpha}_1)) \equiv \frac{s^2 + u_1^2 + s_1^2 + u^2}{2s^2}.$$

The distribution **subtracted with both ISR and FSR** counterterms:

$$d\sigma_1^{\Delta NLO} = d\sigma_1^{NLO} - d\sigma_1^{MCLO} =$$

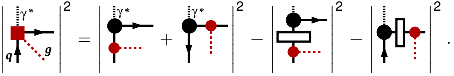
$$= Q_q^2 \alpha_{QED}^2 dt d\varphi \; \frac{1}{t^2} \; \frac{d\psi}{2\pi} \; \delta_y\left(\vec{k}_1\right) \; \frac{C_F \alpha_s}{\pi} \frac{d\bar{\alpha}_1 d\bar{\beta}_1}{\bar{\alpha}_1 \bar{\beta}_1} \; \frac{d\phi_1}{2\pi} \left\{ \bar{\beta}_I \theta_{\bar{\beta}_1 < \bar{\alpha}_1} + \bar{\beta}_F \theta_{\bar{\beta}_1 > \bar{\alpha}_1} \right\},$$

$$\bar{\beta}_I(\bar{\alpha}_1, \bar{\beta}_1) = W(\bar{\alpha}_1, \bar{\beta}_1, y/(1-\bar{\alpha}_1)) - W_0(y/(1-\bar{\alpha}_1)) \bar{P}(1-\bar{\alpha}_1),$$

$$\bar{\beta}_F(\bar{\alpha}_1, \bar{\beta}_1) = W(\bar{\alpha}_1, \bar{\beta}_1, y/(1-\bar{\alpha}_1)) - W_0(y) \bar{P}(1-\bar{\beta}_1),$$

Exclusive NLO correction:

$$\left\{ \bar{\beta}_I \theta_{\bar{\beta}_1 < \bar{\alpha}_1} + \bar{\beta}_F \theta_{\bar{\beta}_1 > \bar{\alpha}_1} \right\} =$$



The above is on the paper, no code tested numerically yet.

Comparison with the MC@NLO scheme:

- The same complete NLO for hard process.
- Soft counterterm for MC and NLO subtraction are the same, hence their difference is zero (nonzero in MCatNLO).
- Positive weight MC events. Narrow weight distribution, WT=1 events possible.
- No plus-distributions $\left(\frac{1}{1-x}\right)_+$ or $\left(\frac{\ln(1-x)}{1-x}\right)_+$ in the NLO corrections (MC weights), all resummed by the LO MC.

Generally, MC@NLO uses existing LO MC PS, we go back and reconstruct PS MC, such that including NLO in the hard process is easier.

Summary and Prospects

- First serious feasibility study of the true NLO exclusive parton shower MC is (almost) complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet in the ladder, plus NLO hard process (DIS).
- Middle range aim: Complete singlet (Q-G transitions) in the ladder.
- Optimize MC weight evaluation (CPU time).
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Interface to standard PDFs
  (or fitting data directly with the Monte Carlo?)
- Extensions towards CCFM/BFKL, quark masses.