CEEX Exponentiation in EQD

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The aim is to summarize briefly on:

- Main features of YFS/CEEX exponentiation in QED
- Examples of resuls relevant for LEP/LC physics program

Based on recent papers by:

S.J., D. Bardin, M. Melles, M. Skrzypek, W. Płaczek, E. Richter-Wąs, T. Riemann,

B.F.L. Ward, and Z. Wąs:

Phys. Rev. D **63**, 113009 (2001),

Eur.Phys. J. **C24**,373, (2002),

Phys.Rev. **D65** 073030, (2002)

These and related slides on http://home.cern.ch/jadach

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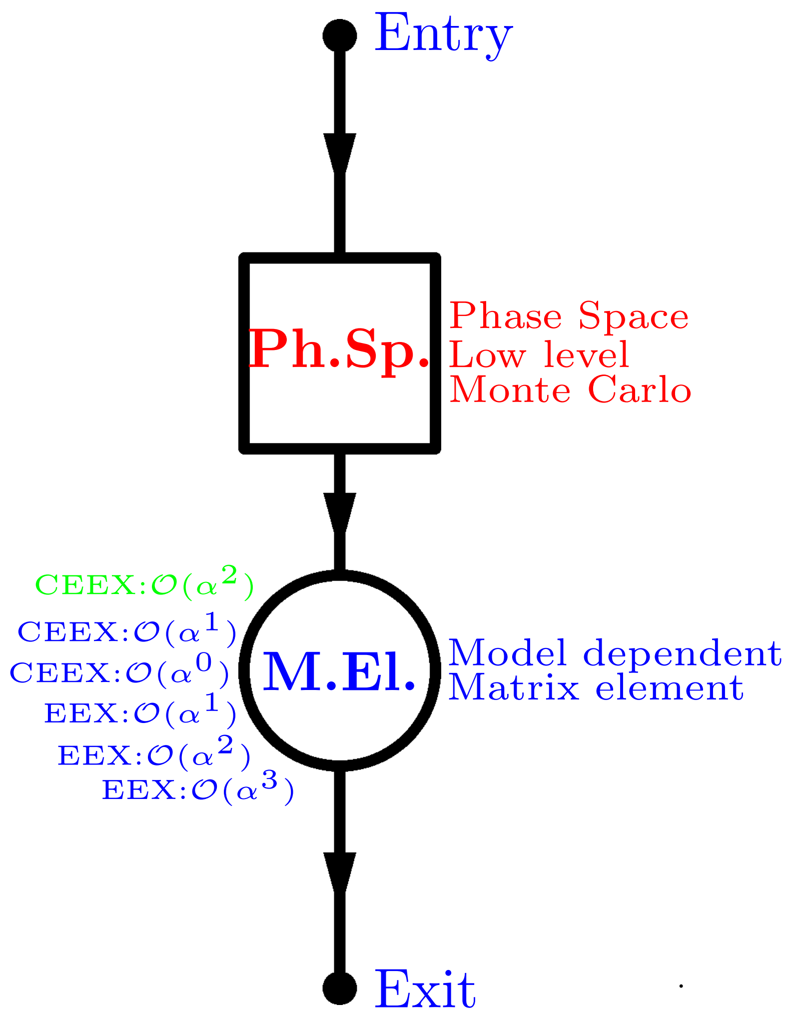
Standard Model calculations for LEP with YFS exponentiation

- $e^+e^- \to f\bar{f} + n\gamma$, $f = \tau, \mu, d, u, s, c$, YFS1 (1987-1989) $\mathcal{O}(\alpha^1)_{exp}$ ISR,
  YFS2∈KORALZ (1989-1990), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ ISR
  YFS3∈KORALZ (1990-1998), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ ISR+FSR
  KKMC (98-02) $\mathcal{O}(\alpha^2 + h.o.LL)_{exp}$ ISR+FSR+Interf. $\boxed{d\sigma/\sigma = 0.2\%}$
- $e^+e^- \to e^+e^- + n\gamma$ for $\theta < 6^\circ$
  BHLUMI 1.x, (1987-1990), $\mathcal{O}(\alpha^1)_{exp}$
  BHLUMI 2.x, (1990-1996), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$ $\boxed{d\sigma/\sigma = 0.07\%}$
- $e^+e^- \to e^+e^- + n\gamma$ for $\theta > 6^\circ$
  BHWIDE (1994-1998), $\mathcal{O}(\alpha^1 + h.o.LL)_{exp}$
- $e^+e^- \to W^+W^- + n\gamma$, $W^\pm \to f\bar{f}$
  KORALW (1994-2001)
- $e^+e^- \to W^+W^- + n\gamma$, $W^\pm \to f\bar{f}$
  YFS3WW (1995-2001), YFS expon. + Leading Pole Approx. $\boxed{d\sigma/\sigma = 0.4\%}$

Typical MC realization: "matrix element × exact phase space" principle



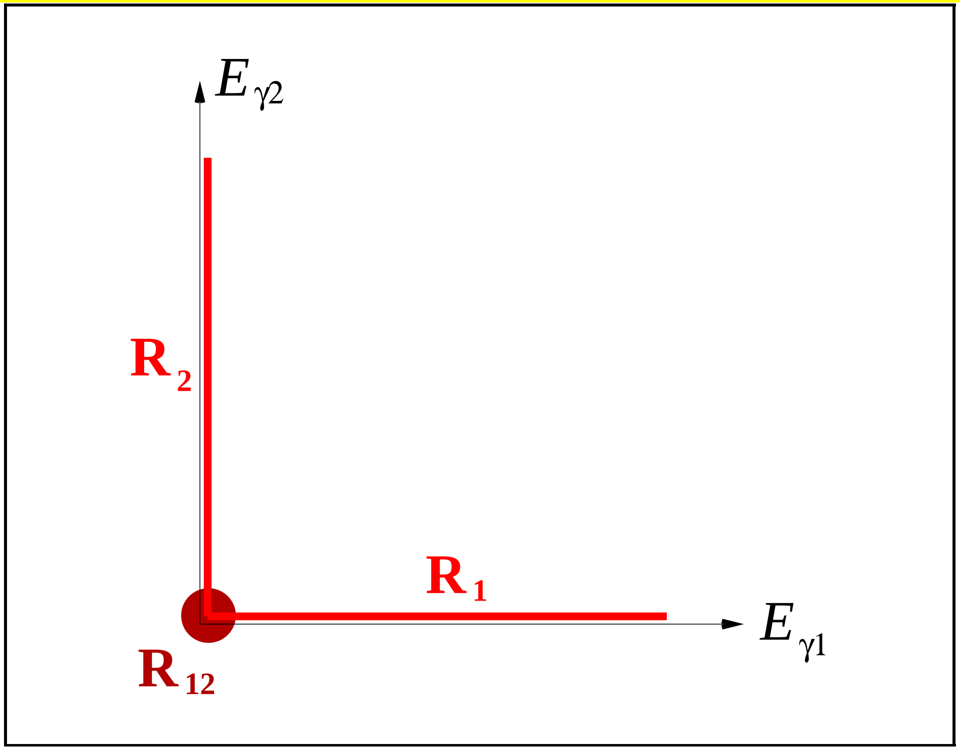
In the Monte Carlo realization it means:

- **Universal exact Phase-space Monte Carlo simulator is a separate module producing "raw events" (with importance sampling)**
- **Library of several types of SM/QED matrix elements provides "model weight" is another independent module (KKMC example is shown)**
- **Tau decays and hadronization come after of course.**

Main steps in YFS exponentiation

- **Reorganization of the perturbative complete $\mathcal{O}(\alpha^\infty)$ series such that IR-finite $\bar\beta$ components are isolated (factorization theorem).**
- **Truncate IR-finite $\bar\beta$s to finite $\mathcal{O}(\alpha^n)$ and calculate them from Feynman diagrams recursively.**

Factorization for overlaping IR divergences for 2γ: $R_{12} \in R_1$ and $R_{12} \in R_2$



$$D_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}) = \bar{\beta}_0(p_{f_1}, p_{f_2}, p_{f_3}, p_{f_4}); \qquad p_{f_1} + p_{f_2} = p_{f_3} + p_{f_4}$$
$$D_1(p_f; k_1) = \bar{\beta}_0(p_f)\tilde{S}(k_1) + \bar{\beta}_1(p_f; k_1); \qquad p_{f_1} + p_{f_2} \neq p_{f_3} + p_{f_4}$$
$$D_2(k_1, k_2) = \bar{\beta}_0\tilde{S}(k_1)\tilde{S}(k_2) + \bar{\beta}_1(k_1)\tilde{S}(k_2) + \bar{\beta}_1(k_2)\tilde{S}(k_1) + \bar{\beta}_2(k_1, k_2).$$

NOTE: $\bar{\beta}_0$ and $\bar{\beta}_1$ used beyond their usual (Born and 1γ) phase space.

A kind of smooth “extrapolation” or “projection” is always necessary.

Recursive, order-by-order definition of IR-finite $\bar{\beta}$s; SUBTRACTIONS!

$$D_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}) = \bar{\beta}_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}),$$
$$D_1(p_f;k_1) = \bar{\beta}_0(p_f)\tilde{S}(k_1) + \bar{\beta}_1(p_f;k_1)$$
$$D_2(k_1,k_2) = \bar{\beta}_0\tilde{S}(k_1)\tilde{S}(k_2) + \bar{\beta}_1(k_1)\tilde{S}(k_2) + \bar{\beta}_1(k_2)\tilde{S}(k_1) + \bar{\beta}_2(k_1,k_2).$$

$$\downarrow$$

$$\bar{\beta}_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}) = D_0(p_{f_1},p_{f_2},p_{f_3},p_{f_4}),$$
$$\bar{\beta}_1(p_f;k_1) = D_1(p_f;k_1) - \bar{\beta}_0(p_f)\tilde{S}(k_1)$$
$$\bar{\beta}_2(k_1,k_2) = D_2(k_1,k_2) - \bar{\beta}_0\tilde{S}(k_1)\tilde{S}(k_2) - \bar{\beta}_1(k_1)\tilde{S}(k_2) + \bar{\beta}_1(k_2)\tilde{S}(k_1).$$

In the practical order-by-order calculations

IR-finite $\bar{\beta}$s are truncated to certain fixed $\mathcal{O}(\alpha^n)$

Classsic EEX/YFS schematically; β's truncated to $\mathcal{O}(\alpha^1)$, example of ISR

$$e^-(p_1,\lambda_1)+e^+(p_2,\lambda_2)\to f(q_1,\lambda'_1)+\bar{f}(q_2,\lambda'_2)+\gamma(k_1,\sigma_1)+\ldots+\gamma(k_n,\sigma_n)$$

$$\sigma=\sum_{n=0}^{\infty}\int_{m_\gamma} d\Phi_{n+2}\, e^{Y(m_\gamma)} D_n(q_1,q_2,k_1,\ldots,k_n)$$

$$D_0=\bar{\beta}_0$$
$$D_1(k_1)=\bar{\beta}_0\tilde{S}(k_1)+\bar{\beta}_1(k_1)$$
$$D_2(k_1,k_2)=\bar{\beta}_0\tilde{S}(k_1)\tilde{S}(k_2)+\bar{\beta}_1(k_1)\tilde{S}(k_2)+\bar{\beta}_1(k_2)\tilde{S}(k_1)$$
$$D_n(k_1,k_2\ldots k_n)=\bar{\beta}_0\tilde{S}(k_1)\tilde{S}(k_2)\ldots\tilde{S}(k_n)+\bar{\beta}_1(k_1)\tilde{S}(k_2)\tilde{S}(k_3)\ldots\tilde{S}(k_n)$$
$$+\tilde{S}(k_1)\bar{\beta}_1(k_2)\tilde{S}(k_3)\ldots\tilde{S}(k_n)+\ldots+\tilde{S}(k_1)\tilde{S}(k_2)\tilde{S}(k_2)\ldots\bar{\beta}_1(k_n)$$

Real soft factors: $\tilde{S}(k)=\sum_\sigma|\mathfrak{s}_\sigma(k)|^2=|\mathfrak{s}_+|^2(k)+|\mathfrak{s}_-(k)|^2=-\frac{\alpha}{\pi}\left(\frac{q_1}{kq_1}-\frac{q_2}{kq_2}\right)^2$

IR-finite building blocks:

$\bar{\beta}_0=\left(e^{-2\alpha\Re B_4}\sum_\lambda|\mathfrak{M}_\lambda^{\rm Born+Virt.}|^2\right)\Big|_{\mathcal{O}(\alpha^1)}$, λ = fermion helicities, σ = photon hel.

$$\bar{\beta}_1(k)=\sum_{\lambda\sigma}|\mathfrak{M}_{\lambda\sigma}^{\rm 1-PHOT}|^2-\sum_\sigma|\mathfrak{s}_\sigma(k)|^2\sum_\lambda|\mathfrak{M}_\lambda^{\rm Born}|^2$$

Everything in terms of $\sum_{spin}|\ldots|^2$! **Distr. <0 possible for hard 2γ.**

New CEEX replaces old EEX, both derived from YFS 1961

EEX= Exclusive EXponentiation, very close to original Yennie-Frautschi-Suura 1961

CEEX = Coherent EXclusive exponentiation, is an extension of YFS

COHERENT = Friendly to Quantum coherence

Coherence among Feynman diags.

Complete $\left|\sum_{diagr.}^{n} \mathcal{M}_i\right|^2$ rather than often incomplete $\sum_{i,j}^{n^2} \mathcal{M}_i \mathcal{M}_j^*$.

Narrow resonances, $\gamma \oplus Z$ exchanges, $t \oplus s$ channels, ISR$\oplus$FSR, Angular ordering, etc.

EEX: KORALZ/YFS2, BHLUMI, YFSWW, KORALZ

CEEX: KKMC

CEEX schematically, the example of ISR $\mathcal{O}(\alpha^1)$

$$e^-(p_1,\lambda_1)+e^+(p_2,\lambda_2)\to f(q_1,\lambda_1')+\bar{f}(q_2,\lambda_2')+\gamma(k_1,\sigma_1)+\ldots+\gamma(k_n,\sigma_n)$$

$$\sigma=\sum_{n=0}^{\infty}\int_{m_\gamma} d\Phi_{n+2}\sum_{\lambda,\sigma_1,\ldots,\sigma_n}\left|e^{\alpha B(m_\gamma)}\mathcal{M}^{\lambda}_{n,\sigma_1,\ldots,\sigma_n}(k_1,\ldots,k_n)\right|^2$$

$\mathcal{M}^{\lambda}_0=\hat{\beta}^{\lambda}_0,\quad \lambda$=fermion helicities,

$$\mathcal{M}^{\lambda}_{1,\sigma_1}(k_1)=\hat{\beta}^{\lambda}_0\mathfrak{s}_{\sigma_1}(k_1)+\hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)$$

$$\mathcal{M}^{\lambda}_{2,\sigma_1,\sigma_2}(k_1,k_2)=\hat{\beta}^{\lambda}_0\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)+\hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)+\hat{\beta}^{\lambda}_{1,\sigma_2}(k_2)\mathfrak{s}_{\sigma_1}(k_1)$$

$$\mathcal{M}^{\lambda}_{n,\sigma_1,\ldots\sigma_n}(k_1,k_2,\ldots k_n)=\hat{\beta}^{\lambda}_0\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)\ldots\mathfrak{s}_{\sigma_n}(k_n)+\hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)\ldots\mathfrak{s}_{\sigma_n}(k_n)$$
$$+\mathfrak{s}_{\sigma_1}(k_1)\hat{\beta}^{\lambda}_{1,\sigma_2}(k_2)\ldots\mathfrak{s}_{\sigma_n}(k_n)+\ldots+\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2)\ldots\hat{\beta}^{\lambda}_{1,\sigma_2}(k_2)$$

$\mathcal{O}(\alpha^1)$ IR-finite building blocks:

$$\hat{\beta}^{\lambda}_0=\left(e^{-\alpha B_4}\mathcal{M}^{\text{Born+Virt.}}_{\lambda}\right)\Big|_{\mathcal{O}(\alpha^1)},$$

$$\hat{\beta}^{\lambda}_{1,\sigma}(k)=\mathcal{M}^{\lambda}_{1,\sigma}(k)-\hat{\beta}^{\lambda}_0\mathfrak{s}_{\sigma}(k)$$

Everything done in terms of $\mathcal{M}$-amplitudes!

Distr. ≥ 0 by construction!

In KKMC the above is done up to $\mathcal{O}(\alpha^2)$ for ISR and FSR

Full Scale CEEX $\mathcal{O}(\alpha^r)$, r=1,2, master formula

Polarized total x-section:

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_n(p_a+p_b; p_c, p_d, k_1, \dots, k_n)\; e^{2\alpha \Re B_4} \sum_{\sigma_i, \lambda, \bar\lambda} \sum_{i,j,l,m=0}^{3}$$

$$\hat\varepsilon_a^i \hat\varepsilon_b^j \sigma^i_{\lambda_a \bar\lambda_a} \sigma^j_{\lambda_b \bar\lambda_b} \mathfrak{M}_n^{(r)} \begin{pmatrix} p\, k_1\, k_2 & \dots & k_n \\ \lambda\, \sigma_1\, \sigma_2 & \dots & \sigma_n \end{pmatrix} \left[\mathfrak{M}_n^{(r)} \begin{pmatrix} p\, k_1\, k_2 & \dots & k_n \\ \bar\lambda\, \sigma_1\, \sigma_2 & \dots & \sigma_n \end{pmatrix} \right]^{\star} \sigma^l_{\bar\lambda_c \lambda_c} \sigma^m_{\bar\lambda_d \lambda_d} \hat h_c^l \hat h_c^m$$

CEEX amplitudes:

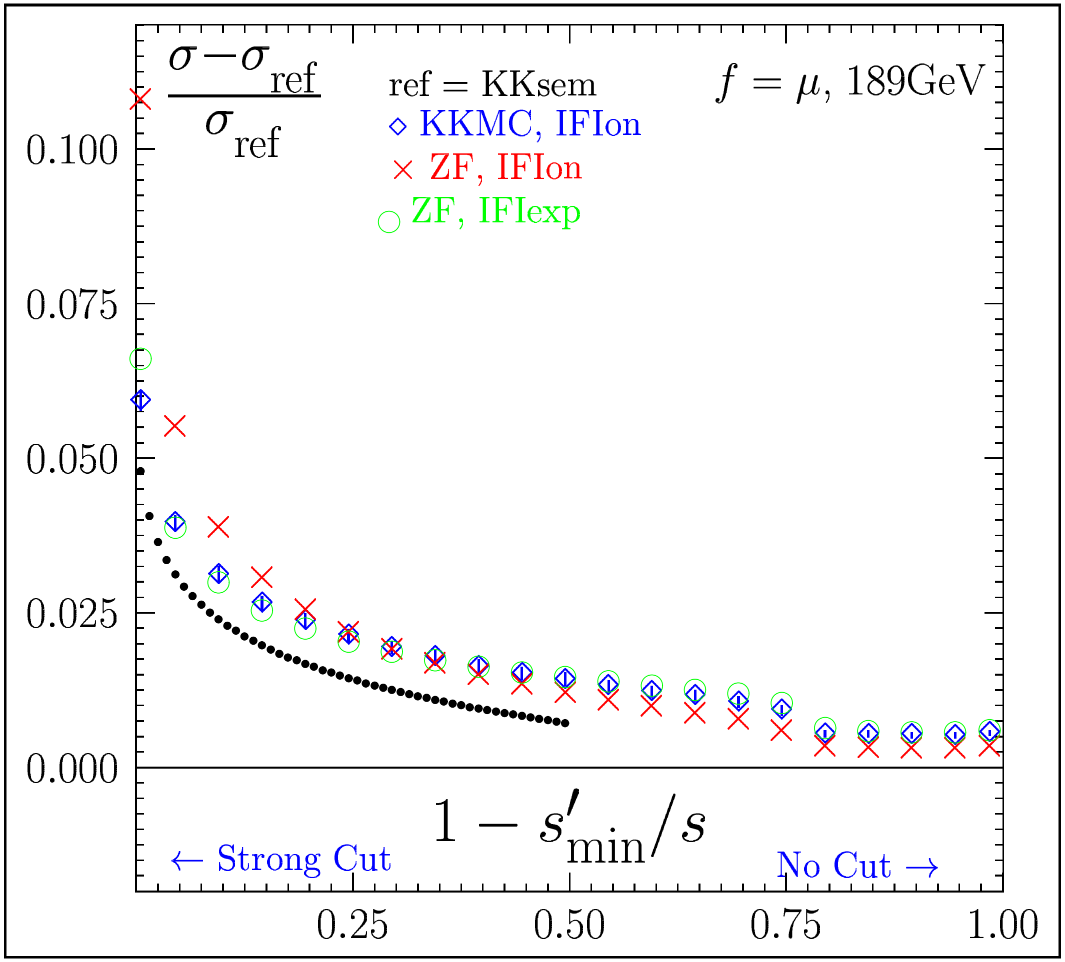
$$\mathfrak{M}_n^{(1)} \begin{pmatrix} p\, k_1 & \dots & k_n \\ \lambda\, \sigma_1 & \dots & \sigma_n \end{pmatrix} = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}} \left\{ \beta_0^{(1)} \begin{pmatrix} p \\ \lambda \end{pmatrix}; X_\wp \Big) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{1\{\wp_j\}} \begin{pmatrix} p\, k_j \\ \lambda\, \sigma_j \end{pmatrix}; X_\wp \Big)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} \right\}$$

$$\mathfrak{M}_n^{(2)} \begin{pmatrix} p\, k_1 & \dots & k_n \\ \lambda\, \sigma_1 & \dots & \sigma_n \end{pmatrix} = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}}$$

$$\times \left\{ \beta_0^{(2)} \begin{pmatrix} p \\ \lambda \end{pmatrix}; X_\wp \Big) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{2\{\wp_j\}} \begin{pmatrix} p\, k_j \\ \lambda\, \sigma_j \end{pmatrix}; X_\wp \Big)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} + \sum_{1 \le j < l \le n} \frac{\beta^{(2)}_{2\{\wp_j, \wp_l\}} \begin{pmatrix} p\, k_j\, k_l \\ \lambda\, \sigma_j\, \sigma_l \end{pmatrix}; X_\wp \Big)}{\mathfrak{s}_{[j]}^{\{\wp_j\}} \mathfrak{s}_{[l]}^{\{\wp_l\}}} \right\}$$

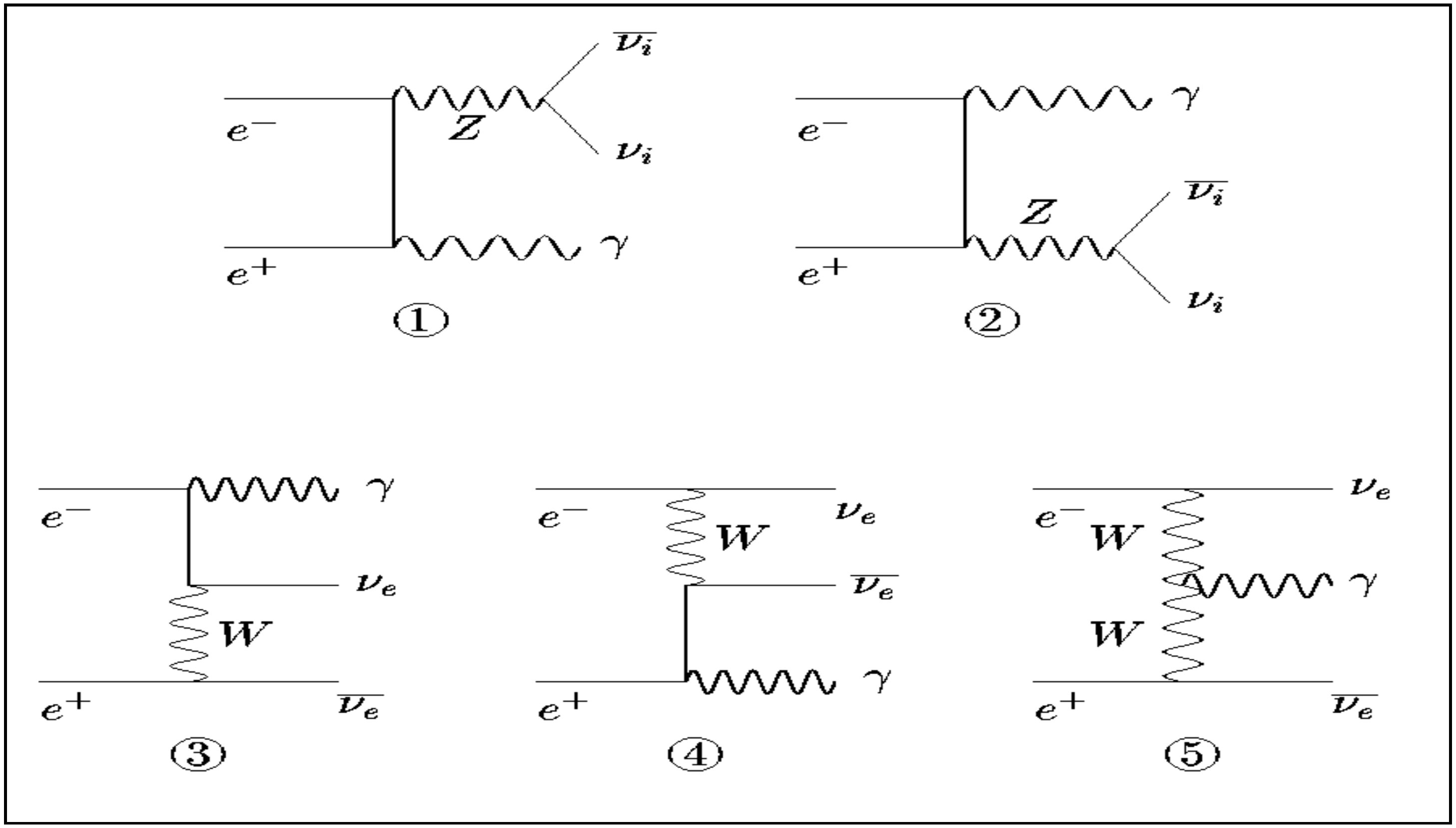
For details see Phys. Rev. D 63, 113009 (2001).

KK MC vs. Zfitter for ISR+FSR, including IFI=ISR⊗FSR; $d\sigma/\sigma = 0.2\%$



Comparison of KKMC and Zfitter. ISR of Zfitter is based on the $\mathcal{O}(\alpha^2)$ result by Berends, Burgers, VanNeerven (1984), while KKMC is totally independent!

Improvements of CEEX in KKMC for $e^+e^- \to \nu\bar{\nu}\gamma$ process

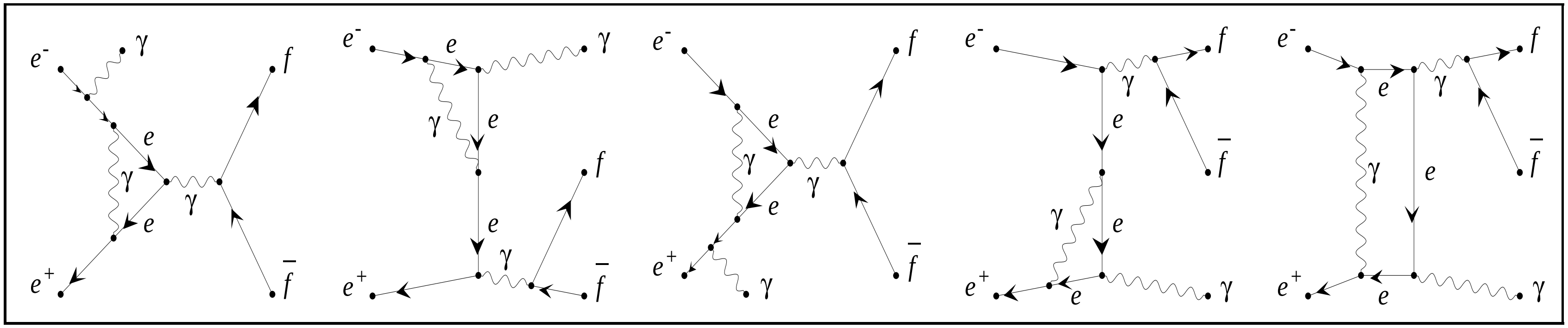


- $\mathcal{KK}$ MC with CEEX matrix element is now extended to the neutrino mode. (Eur.Phys.J.C24:373-383,2002, hep-ph/0110371)
- It is a replacement for KORALZ older program.
- Useful for LEP and 1-st step towards LC.

Improvements of CEEX in KKMC for $e^+e^- \to \nu\bar{\nu}\gamma$ process

- **The systematic error is estimated to be 1.3% for $\nu_e\bar{\nu}_e\gamma$ and 0.8% for $\nu_\mu\bar{\nu}_\mu\gamma$ and $\nu_\tau\bar{\nu}_\tau\gamma$.**
- **For observables with two observed photons we estimate the uncertainty to be about 5%.**
- **These new improved results were obtained thanks to the inclusion of non-photonic electroweak corrections of the** ZFITTER **package and due to newly constructed, exact, single and double emission photon amplitudes in the $\mathcal{KK}$ MC for the contribution with the t-channel W exchange.**
- **The virtual corrections for the W exchange are at present introduced in the approximated form. The exponentiation scheme CEEX is the same as in the original $\mathcal{KK}$ MC program**

Exact Differential $\mathcal{O}(\alpha^2)$ Results for Hard Bremsstrahlung in $e^+e^- \to 2f$

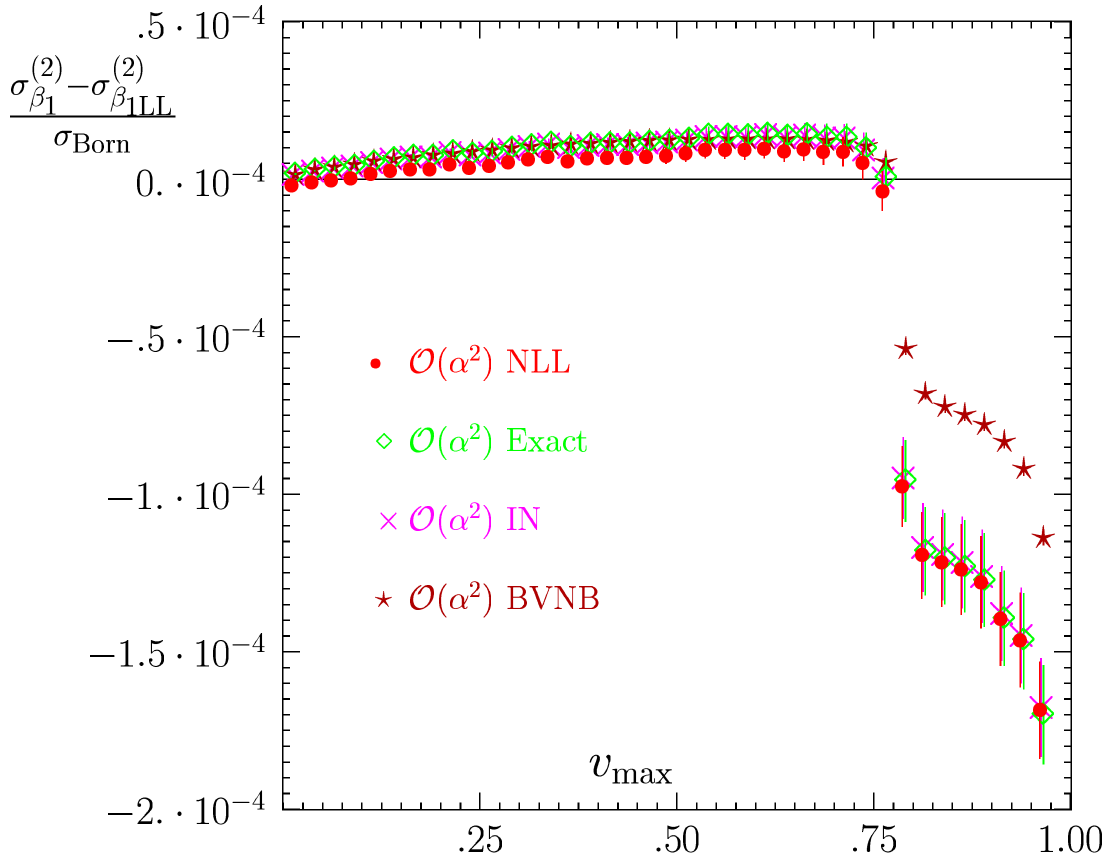


Phys.Rev.D65:073030,2002; hep-ph/0109279

- **Fully differential results for $2f + 1\gamma_{virt} + 1\gamma_{real}$ checked against Igarashi and Nakazawa**
- **Partly integrated results checked against Berends, Burgers and VanNeerven**
- **Important component for any eaxact $\mathcal{O}(\alpha^2)$ exponentiated calculation for $e^+e^- \to 2f$ is now available.**

Similar works were also completed by Karlsruhe group recently, hep-ph/0204283 .

Exact Differential $\mathcal{O}(\alpha^2)$ Results for Hard Bremsstrahlung in $e^+e^- \to 2f$



For $v < 0.9$ the agreement to within $0.5 \cdot 10^{-4}$ reached

Conclusions

- **YFS inspired EEX and CEEX schemes are succesful examples of the Monte Carlos based directly on the factorization theorem (albeit for IR soft case for abelian QED only).**
- **Work well in practice: KORALZ, BHLUMI, YWSWW3, BHWIDE, KK MC.**
- **Extension (as far as possible) to all collinear singularities would be very desirable and practicaly important!**
- **The KKMC program is extended to neutrino channel**
- **Missing fully differential $2f + 1\gamma_{virt} + 1\gamma_{real}$ distributions for $\mathcal{O}(\alpha^2)$ CEEX are now available**