Towards NLO Parton Shower MC

S. JADACH

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What is NLO parton shower?

As a warm-up consider: **What is the LO parton shower?**

- The LO parton shower MC is built using LO class evolution kernels and/or LO PDFs within well defined collinear factorization scheme
- LO PS MC implements LO DGLAP evolution for semi-inclusive distributions (structure functions etc.).
- In case hard process is corrected to the NLO level (N+LO), LO PS MC *desirably* encapsulates all collinear/soft singularities of the hard process ME, in the exclusive form.
- In the N+LO schemes ONE parton originally generated within the LO PS MC gets promoted to the hard process.

What is NLO parton shower?

Now everything **one order higher than LO**:

- The NLO parton shower MC is built using NLO class evolution kernels and/or NLO PDFs.
- NLO PS MC implements NLO DGLAP evolution for certain semi-inclusive distributions (structure functions etc).
- For hard process corrected to the N^2LO level (N+NLO), it is desirable that NLO PS MC encapsulates all collinear/soft singularities of the N^2LO hard process ME in the exclusive form. (Complete coverage of ph.sp. for hard. proc. by PS MC is a separate issue.)
- In the N+NLO scheme up to TWO partons originally generated within the NLO PS MC get promoted into the hard process,

Problems and solutions

- NLO kernels have to be recalculated in the exclusive form.
  - We have recalculated all NLO kernels using Curci-Furmanski-Petronzio (CFP) scheme – explicit diagrammatic calculation in axial gauge (also Ellis+Voghesang, Kunst+Heinrich).
  - Technical improvements were proposed (Skrzypek+Gituliar)
- LO parton shower may miss some phase space regions which are present in NLO kernels/evolution, like $q \to qG^*$, $G^* \to GG$ splitting
  - Luckily, modern LO PS MCs like HERWIG++, Sherpa, Pythia8 kindly populate parts of ph.space needed for the NLO PS.
- Introducing complete NLO real and virtual corrections into PS MC in the exclusive form, in accordance with the collinear factoriz. theorems (CFP), is the main problem, theoretically and practically.
  - Theoretical framework CFP-based is being formulated and tested,
  - 3 methods of practical implementation of NLO corrections in the PS MC were formulated and tested. One of them quite promissing.

Remarks on NLO kernel re-calculation

- Why CFP? Defined precisely, $|\text{ME}|^2 \times$Ph.Sp., NLO-practical.
- Inclusive $\overline{MS}$ kernels we have reproduced listed/exploited. Exclusive 2-real and 1real+1virtual distributions, before the phase space integration, are now available.
- CFP was modified in order to eliminate spurious $1/\varepsilon^3$ poles obscuring relation to MC at $d = 4$ dimensions. The so-called NPV prescription by Skrzypek and Gituliar, published recently.
- For subsets of diagrams in 2-real parton contributions, soft gluon limit was analyzed carefully. Expected gauge cancellations found.
- In CFP NLO kernel is extracted as coefficient of $1/\varepsilon$. An alternative method using derivative $\partial/\partial(\ln \mu^2)$ was tested.
- $\overline{MS}$ scheme produces technical artifact $\sim \varepsilon/\varepsilon^2$, which are source of the problems in the MC implementation of NLO corrections. These terms were classified and their role was analyzed.

Theoretical framework of PS MC: Collinear Factorization

- Collinear factorization in a nutshell:

$$F_{bare}(q_h/\mu,\varepsilon) = \frac{\sigma_{Bare}}{\sigma_{Born}} = \prod_{Ladders} C^{(\infty)}\left(\alpha, \frac{q_h}{\mu}\right) \otimes \Gamma^{(\infty)}_{ladder}(\alpha,\varepsilon)$$

  Operator $\otimes$ is in lightcone x and parton type,
  Γ is inclusive, C can be kept unintegrated/exclusive.

$$\text{Case LO}: \quad F^{(1)}_{bare}(q_h/\mu,\varepsilon) = [\mathbb{1} + C^{[1]}(\alpha, q_h/\mu)] \otimes [\mathbb{1} + \Gamma^{[1]}(\alpha,\varepsilon)]$$

$$\text{Case NLO}: \quad F^{(2)}_{bare}(q_h/\mu,\varepsilon) = [\mathbb{1} + C^{[1]} + C^{[2]}] \otimes [\mathbb{1} + \Gamma^{[1]} + \Gamma^{[2]}]$$

- In physical diff. distributions: $\Gamma \rightarrow \mathrm{PDF}$. LO example:

$$F_{Phys.} = [\mathbb{1}+C^{[1]}(\alpha, q_h/\mu)]\otimes \mathrm{PDF}(\mu), \quad C^{[1]}(q_h/\mu) \equiv F^{[1]}_{bare}(q_h/\mu,\varepsilon) - \Gamma^{[1]}(\varepsilon)$$

  $F_{Phys.}$ factor. scheme independent; both C and PDFs are dependent:

$$\Gamma^{[1]}(\varepsilon) \rightarrow \Gamma^{[1]} + \Delta\Gamma^{[1]}, \quad C^{[1]} \rightarrow C^{[1]} - \Delta C^{[1]}, \quad \Delta C^{[1]} = -\Delta\Gamma^{[1]}.$$

- Evolution of F and/or PDFs and evolution kernels:

$$\frac{\partial}{\partial \ln \mu^2} F(\mu) = P \otimes F(\mu), \quad P = \alpha P^{[0]} + \alpha^2 P^{[1]} + \ldots = \mathrm{Res}_1 \Gamma(\varepsilon) = \frac{\partial \ln_{\otimes} C(q/\mu)}{\partial \ln \mu^2}$$

Collinear Factorization – Fixed order calculations

- Fixed order calculation, like MCFM:

$$F(q_h) = [\mathbb{1} + C^{[1]}]_J \otimes \mathrm{PDF}(\mu), \quad C^{[1]} \equiv [F^{[1]}_{bare}(q_h/\mu, \varepsilon) - \Gamma^{[1]}(\varepsilon)]_{q_h=\mu}$$

$[...]_J$ means experimental acceptance $J(x, y)$ kept in integrand.

- Typical example: ISR gluonstrahlung part of DIS, def. $y = q/q_h \in (1, 0)$:

$$C^{[1]}(z, y) = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \frac{C_F\alpha}{\pi}\left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+ + \beta(z, y) + \delta_{y=0}\Sigma(z)$$

$$\beta(z, y) = |\mathrm{ME}_{exact}|^2 - \frac{C_F\alpha}{\pi}\frac{1}{y}\frac{\bar{P}(z)}{1-z}, \qquad \Sigma(z) = \frac{C_F\alpha}{\pi}\left(\frac{\bar{P}(z)}{1-z}\ln\frac{(1-z)^2}{z}\right)_+$$

where $\bar{P}(z) = (1-z)P_{qq}(z) = (1+z^2)/2$.

- Soft-collinear subtraction technique (eg. Catani-Seymour) often used:

$$C^{[1]} = [F^{[1]}_{bare} - C_{SC}]_{d=4} + [C_{SC} - \Gamma^{[1]}]_{d\neq 4}, \quad C_{SC}(z, y) = \frac{C_F\alpha}{\pi}\frac{1}{y^{1-2\varepsilon}}\left(\frac{\bar{P}(z)}{1-z}\right)_+$$

LO and N+LO parton shower MC

- Pure LO parton shower MC, again *ep* with single ISR ladder:

$$F(q_h) = G_{q_0 \to q_h} \otimes \mathrm{PDF}_{\mu=q_0 \simeq 1 GeV}$$

$$G_{q_0 \to q_h} = \exp_{y-ordering} \left\{ \int_0^1 dy \left(\frac{1}{y}\right)_+ \int_0^{2\pi} d\phi \int_0^1 dz\, \frac{C_F \alpha}{\pi} (P^{[0]}(z))_+ \right\}$$

where $y = q/q_h$ and $(1/y)_+$ regulated using $y > \Delta = q_0/q_h$.

- The above is forward evol. Backward evolution PS MC starts from q_h:

$$F(q_h) = \mathrm{PDF}_{\mu=q_h} \otimes (G_{q_0 \to q_h})^{-1}$$

- N+LO parton shower (POWHEG or MCatNLO) is schematically:

$$F(q_h) = [\mathbb{1} + \tilde{C}^{[1]}] \otimes G_{q_0 \to q_h} \otimes \mathrm{PDF}_{\mu=q_0 \simeq 1 GeV}$$

where in $C^{[1]} \to \tilde{C}^{[1]}$ LO MC part is subtracted, to omit 2-counting:

$$C^{[1]}(z,y) = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \frac{C_F \alpha}{\pi} \left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+ + \beta(z,y) + \delta_{y=0}\Sigma(z)$$

$$\tilde{C}^{[1]} = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \beta(z,y) + \delta_{y=0}\Sigma(z)$$

but the peculiar $\Sigma(z)$, $\overline{\text{artifact}}$ of $\overline{MS}$, due to ε/ε terms remains!

KRLnlo variant of N+LO parton shower MC

A simpler alternative to POWHEG or MC@NLO

- The backward evolution PS MC plus NLO-corrected hard process

$$F(q_h) = [\mathbb{1} + \tilde{C}^{[1]}] \otimes \mathrm{PDF}^{\overline{MS}}_{\mu=q_h} \otimes (G_{q_0\to q_h})^{-1}$$

$$\tilde{C}^{[1]} = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \beta(z,y) + \delta_{y=0}\Sigma(z)$$

- we reorganize in the following way, KRKnlo method:

$$F(q_h) = [\mathbb{1} + \bar{C}^{[1]}] \otimes \mathrm{PDF}^{\mathrm{MC}}_{\mu=q_h} \otimes (G_{q_0\to q_h})^{-1},$$

$$\bar{C}^{[1]}(y,z) = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \beta(z,y),$$

- where $\mathrm{PDF}^{\overline{MS}}$ is translated to MC factorization scheme outside PS MC:

$$\mathrm{PDF}^{\mathrm{MC}}(\mu) \equiv (\mathbb{1} - \Sigma) \otimes \mathrm{MC}^{\overline{MS}}(\mu)$$

- In reality Σ is matrix in flavour space and mixes $q \leftrightarrow G \leftrightarrow \bar{q}$.
  Its element are fixed from inspecting at least two processes.
- **It was tested for DY process, see later on...**

Important observation

to be generalized to the N+NLO level

- On may notice that collecting all previous steps, we have got:

$$\bar{C}^{[1]} = F^{[1]}_{bare}(\varepsilon)|_{q_h=\mu} - C^{\rm MC}_{\rm SC}(\varepsilon) = (1+\Delta_{SV})\mathbb{1} + \beta(z,y).$$

$$C^{\rm MC}_{\rm SC}(y,z,\varepsilon) = \delta_{y=0}\Gamma^{[1]}(z,\varepsilon) + \delta_{y=0}\Sigma(z) + \frac{C_F\alpha}{\pi}\left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+,$$

- where $C^{\rm MC}_{\rm SC}(\varepsilon)$ is the 1-st order part
of the evolution operator of the LO PS MC in $d = 4+2\varepsilon$:

$$G^{d=4+2\varepsilon}_{q_0\to q_h} = \mathbb{1} + G^{[1]}(\varepsilon) + ..., \qquad C^{\rm MC}_{\rm SC}(\varepsilon) = G^{[1]}(\varepsilon) \;!!!$$

- $C^{\rm MC}_{\rm SC}$ may be also employed/tested as a soft-collinear counterterm
within the NLO fixed order calculation (MCFM-style), with PDFs in the MC scheme:

$$F(q_h) = \left[\mathbb{1} + \bar{C}^{[1]} + \frac{C_F\alpha}{\pi}\left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+\right]_J \otimes {\rm PDF}^{\rm MC}|_{\mu=q_h},$$

This was tested in the modified version of MCFM for DY process.

N+NLO parton shower MC for the 1st time:

- Use collinear factoriz. of Curci-Furmanski-Petronzio (CFP) as a basis. The 2-nd order version reads:

$$F^{(2)}_{bare}(q_h,\varepsilon) = C^{(2)}(q_h/\mu) \otimes \prod_{Ladders} \Gamma^{(2)}_L(\varepsilon), \quad C^{(2)} = F^{(2)}_{bare} \otimes \prod_L (\Gamma^{(2)}_L)^{-1}$$

  and exploit the observation noticed in the previous N+LO case.

- Fixed order N^2LO with collinear $\overline{MS}$ PDFs (one ladder) is now:

$$F^{(2)}_{phys.}(q_h) = C^{(2)}|_{q_h=\mu} \otimes \mathrm{PDF}^{\overline{MS}}(\mu)$$

- Generalizing N+LO observation, define/use MC distribution truncated to 2-nd order $G^{(2)}_{MC}$ in $d = 4+\varepsilon$ as a soft-collinear countertem:

$$F^{(2)}(q_h) = \{F^{(2)}_{bare} \otimes (G^{(2)}_{MC})^{-1}\}_{d=4} \otimes \{G^{(2)}_{MC}(\varepsilon) \otimes (\Gamma^{(2)}(\varepsilon))^{-1}\} \otimes \mathrm{PDF}^{\overline{MS}}(\mu)$$

- The key point is to construct the NLO evolution operator G_{MC} such that
  - $G^{(\infty)}_{MC,D=4}$ represents NLO parton shower MC (single ladder) and
  - $G^{(2)}_{MC,d=4+2\varepsilon}$ encapsulates ALL of collinear and soft singularities in the CFP construction of the NLO $\overline{MS}$ evolution kernel $P^{(2)}(z)$.

NLO ladder in N+NLO parton shower MC

- Explicit simple example of NLO evolution operator G_{MC} in $d = 4$, again for the gluonstrahlung branch (extension to $d = 4 + 2\varepsilon$ is trivial).

$$dG^{(2)}_{MC,d=4} = \mathbb{1} + dy_1 dz_1\; g^{[1]}_{MC}(y_1, z_1)\big(1 + V^{[1]}(z_1)\big)$$
$$+ dy_1 dz_1 dy_2 dz_2 \theta_{y_2>y_1} \big[g^{[1]}_{MC}(y_1, z_1) g^{[1]}_{MC}(y_2, z_2) + \beta^{[1]}(y_2/y_1, z_2/z_1)\big]\big\}$$
$$g^{[1]}_{MC}(y, z) = \frac{C_F \alpha}{\pi} \Big(\frac{1}{y}\Big)_+ \Big(\frac{\bar{P}(z)}{1-z}\Big)_+,$$

  where LO component $g^{[1]}_{MC}$ is already known from N+LO exercise.
- NLO corrections $V^{[1]}(z)$ and $\beta^{[1]}(y, z)$ should come from comparing/matching/analyzing $G^{(2)}_{MC,d\neq 4}$ and elements of CFP scheme.
- The above matching procedure is formulated, still getting consolidated.
- Basic elements of CFP, see next slide...

Elements of the CFP (EGMPR) scheme

- CFP factorization formula for single ladder with two-particle-irreducible (2PI) kernels K_0 in the axial gauge:

$$F = C_0 \cdot \frac{1}{1-K_0} = C_0 \cdot \sum_{n=0} K_0^n.$$

- It is reorganized using projection operator $\mathbb{P} = P_{spin}\, P_{kin}\, \mathrm{PP}$, with kinematic P_{kin}, P_{spin} spin parts and PP extracting pole part $\sim 1/\epsilon^k$.

$$F = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right) = C_0 \cdot \frac{1}{1-[(1-\mathbb{P})K_0]} \otimes \frac{1}{1-\left\{\mathbb{P}K_0 \cdot \frac{1}{1-[(1-\mathbb{P})K_0]}\right\}_{\otimes}}.$$

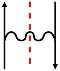
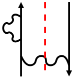
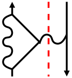
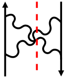
- Second order truncation exploiting 2-nd order $K_0^{(2)} = K_0^{[1]} + K_0^{[2]}$:

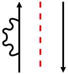
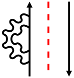
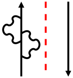
$$\Gamma^{(2)} = \mathbb{1} + \mathbb{P}K_0^{(2)} + \mathbb{P}(K_0^{[1]} \cdot (1-\mathbb{P})K_0^{[1]}) + (\mathbb{P}K_0^{(1)}) \otimes (\mathbb{P}K_0^{(1)})$$

- An example of the diagrammatic content of $K_0^{(2)} = K_0^{[1]} + K_0^{[2]}$ for gluonstrahlung is shown on the next slide...

Contributions to example 2PI $\sim C_F^2$ kernel $K_0(q \to q)$:

$$K_0 = K_0^{[1]} + K_0^{[2]},$$

$K_0^{[1]} =$  , $K_0^{[2]} =$  $+$  $+$ 

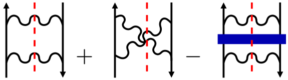
$Z_F = 1 + Z_F^{[1]} + Z_F^{[2]},$ $Z_F^{[1]} =$  , $Z_F^{[2]} =$  $+$ 

Determining NLO 2-real corrections $\beta^{[1]}(y,z)$ for the MC ladder

- Within the same gluonstrahlung example, determination of $\beta^{[1](y,z)}$ is rather simple:

$$\beta^{[1]}(y_2/y_1, z_2/z_1) = |\mathrm{ME}_{2r}|^2 - g^{[1]}_{MC}(y_2, z_2) g^{[1]}_{MC}(y_1, z_1).$$

- The same RHS diagrammatically:



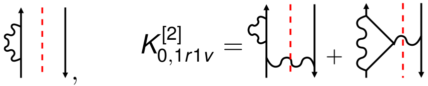
- NB. Such an internal subtraction of the $(\mathrm{LO}_{\mathrm{MC}})^2$ contribution is necessary only for a small subset of NLO diagrams.

Determining 1-real 1-virtual NLO corrections $V^{[1]}(z)$

is most tricky point...

- 1real + 1virtual contribution to $V^{[1]}(z)$ comes from diagrams:

$Z_F^{[1]} =$, $K_{0,1r1v}^{[2]} =$ +



all the time $\sim C_F^2$ glunstrahlung example...

- Determination of $V^{[1]}(z)$ is not easy – it involves several issues:
  - Extracting $\Gamma(\varepsilon)$ from 1r1v part of MC in $d = 4 + 2\varepsilon$ requires
    (i) either extension of CFP subtraction recipe or
    (ii) adjusting IR cut-off $(1 - z) < \delta$ in such that some terms disappear.
  - CFP subtraction have to be done separately for the virtual Sudakov formfactor.
  - In principle $V^{[1]}$ could also depend on y variable.
    This dependence in fact materializes from the UV subtraction.
    However, such terms contribute only pure $1/\varepsilon^2$ to Γ (pure $(\mathrm{LO_{MC}})^2$ in finite part)
    and have to be removed, to avoid double counting with the exponentiated LO MC.
  - The presence/absence of $\sim \Delta_{\mathrm{CFP}}$ has to be decided.
- Whichever way we find: $\bar{P}(z) \equiv (1 + z^2)/2$

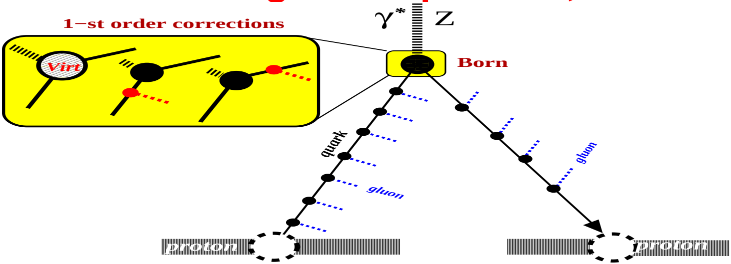
$$V^{[1]}(z) = -\frac{1}{2}\frac{\bar{P}(z)}{1-z}\ln(z)\ln(1-z) + \frac{1}{2}\frac{\bar{P}(z)}{1-z}\mathrm{Li}_2(1-z) + \frac{z}{8}.$$

Examples of numerical implementations

Examples of numerical implementation results for:

- NLO corrections to hard process,
  KRKnlo alternative to MCatNLO and/or POWHEG.
- NLO corrections in the ladder,
  for NLO parton shower MC + NNLO hard process.

N+LO correcting HARD process, KRKnlo method



NLO correction introduced using simple positive MC weight:

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Fj})}{\bar{P}(z_{Fj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Bj})}{\bar{P}(z_{Bj})\; d\sigma_B(\hat{s}, \hat{\theta})/d\Omega}, \qquad \bar{P}(z) \equiv \frac{1+z^2}{2}$$

The IR/Col.-finite real emission part is

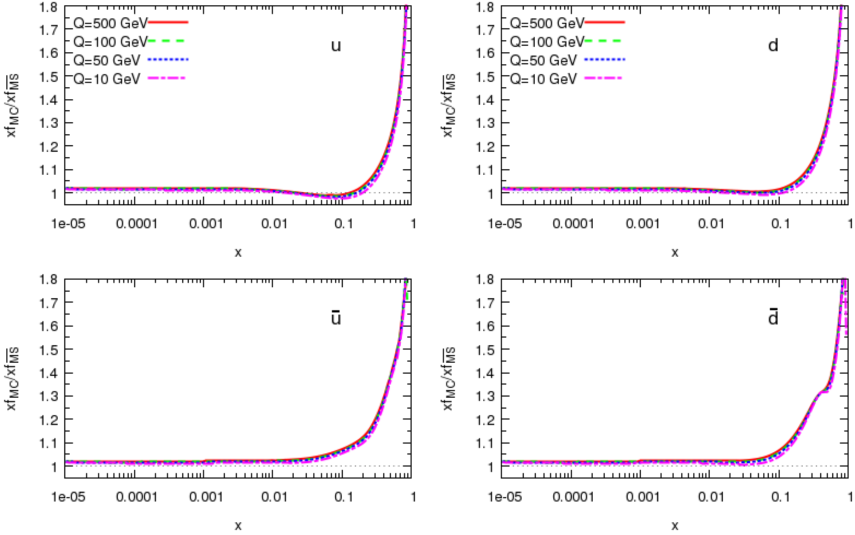
$$\tilde{\beta}_1(\hat{p}_F, \hat{p}_B; q_1, q_2, k) = \left[\frac{(1-\alpha)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_{F1}) + \frac{(1-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_{B2})\right]$$
$$-\,\theta_{\alpha>\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}) - \theta_{\alpha<\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}),$$

the kinematics independent virtual+soft correction is $\Delta_{V+S} = \frac{C_F\alpha_S}{\pi}\left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F\alpha_S}{\pi}\frac{1}{2}$

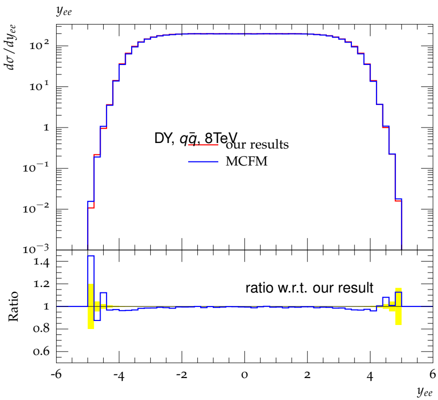
Terms like $\left(\frac{f(z)}{1-z}\right)_+$ in virt. corrections are completely absent!

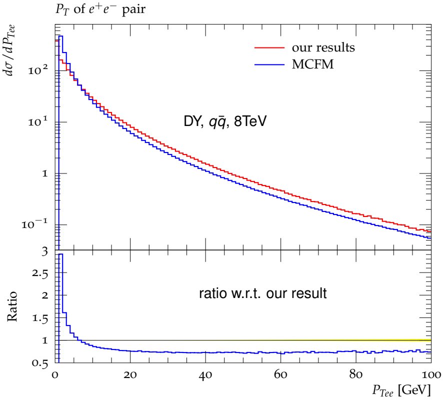
4. Redefined PDFs: $\overline{MS} \rightarrow$ MC scheme

Ratios with respect to standard $\overline{MS}$ PDFs for light quarks.



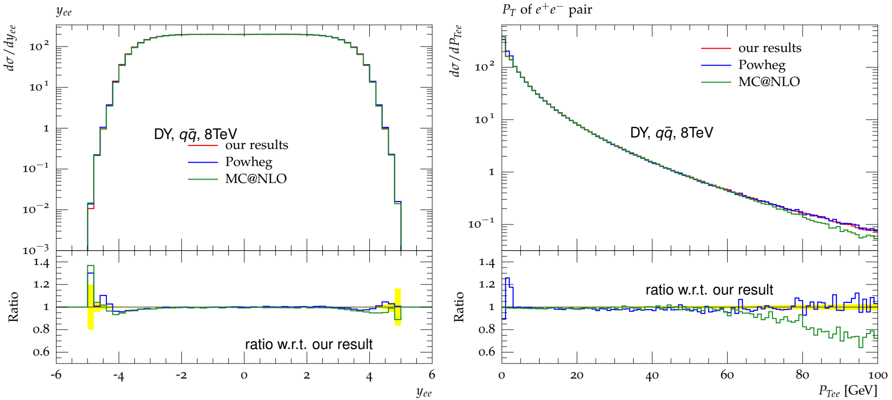
pT and rapidity distributions, KRKnlo vs MCFM





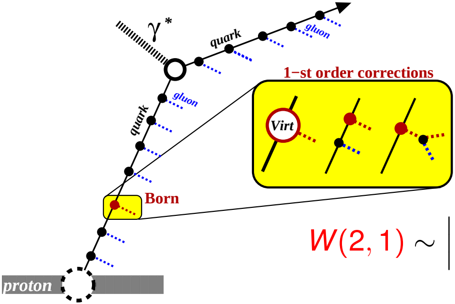
- Our KRKnlo on top of Sherpa LO MC, $q\bar{q}$ channel only.
- y_Z distribution from KRKnlo agrees with MCFM at NLO.
- p_T distribution suppressed at low p_T due to Sudakov.
- Virtual correction spread over a range of p_T.

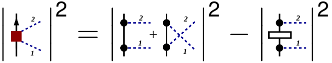
KRKnlo vs. POWHEG and MC@NLO



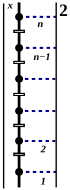
- y_Z and p_T distributions very close to POWHEG (difference at low p_T due to slightly different evolution variable)
- y_Z very close to MC@NLO, same for low and intermediate p_T (differences for the tail of p_T distributions due to higher orders as expected)
- The above is for $q\bar{q}$ channel. Results for qG channel still validated.

NLO-corrected middle-of-the-ladder kernel, C_F^2

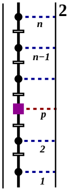


$W(2,1) \sim$ 

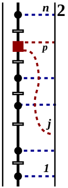
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p=1}^{n}$$



$$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$



$$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int\limits_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}.$$

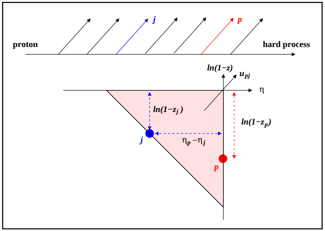
Concept proof using simple Markovian LO PS MC

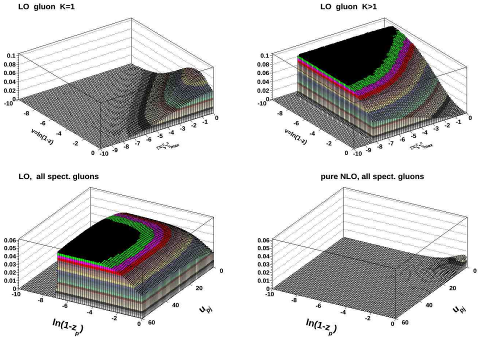
and NLO multiplicative weight

To speeds up considerably MC NLO weight calculation,

for a given p, only one j-term in $\sum_{p=1}^{n} \sum_{j=1}^{p-1}$ is kept, the one with maximum u_{pj}.

$u_{pj} = |\eta_p - \eta_j| + \lambda \ln(1 - z_j)$, $\lambda \sim 1$ used for "u-ordering" in the middle of the ladder, where η is rapidity, z is conventional lightcone variable.

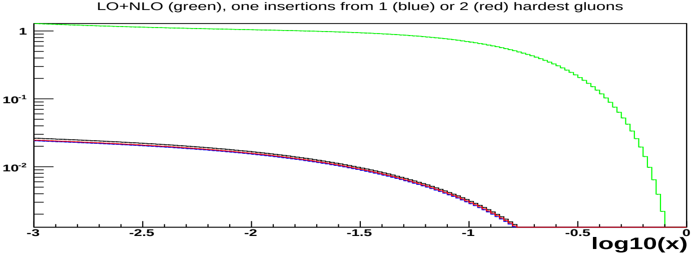


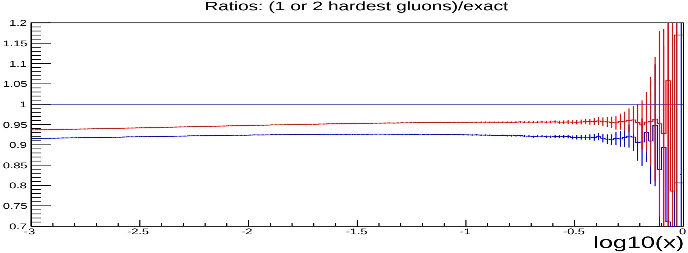


NLO corr. to evolution comes from product of two bottom distributios or alternatively...

Repetition of test for NLO-corrected ladder

NEW: NLO contrib. to 1 kernel, 1 and 2 gluons with max. kT





This difference $\sim$ 15% is formally the NNLO/NLO class. (Faster in CPU time).

Summary

- **An alternative (to MC@NLO or POWHEG) scenario for NLO-corrected hard proc. and LO PSMC is worked out.**
- **Parton shower MC implementing complete NLO DGLAP in the ladders in exclusive way is progressin.**
- General aim: N+NLO= NLO ladder + NNLO hard process, (but LO ladder + NLO hard proc. optimized first!)
- Comming application: high quality QCD+EW+QED MC with hard process like $W/Z/H$ boson production + decay.
- Potential gains from new QCD methods are:
  – reducing h.o. QCD uncertainties
  – easier implementation of NLO and NNLO corrections to hard process.
  – better environment for low x resumm. (BFKL, CCFM),
  – and more...