Collinear factorization in CCD for pedestrians and not only

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October 25th, 2010

More material on http://jadach.web.cern.ch/

Abstract (plan of the talk)

- I am going to explain in a simple pictorial way the essential idea and practical toolbox of the collinear factorization in QCD:
  (i) factorizing hard process from the hadron part,
  (ii) defining PDF (parton distribution function),
  (iii) notion of universality, factorization scheme dependence.
- This factorization, together with the earlier understanding from QED of the soft photon/gluon limit resummation, is the cornerstone of all practical calculations of the QCD effects in the hadron colliders, like LHC and Tevatron.
- For the future analysis of data from both colliners it will be critical to put theoretical pQCD calculation at the new unprecedented precision level, both for the Standard Model processes and for any New Physic phenomena.
- In the second part of my talk I shall briefly outline newly developed more powerful method of the collinear factorization in perturbative QCD, which is implementable using Stochastic Simulation methods (Monte Carlo), and therefore opening new avenues for the practical QCD calculations.

PART I

Collinear Factorization Theorems in perturbative QCD for pedestrians

Collinear factorization in QCD: why?

Comparing QCD with Quantum Electrodynamics (QED):

- in QED infrared (IR) infinities (divergences) from integrating over small photon energies are removed (regularized) by providing photon temporarily with a small mass, and later on, while combining real and virtual Feynman diagrams they always cancel; photon mass can be finally reset to zero. (Cancellations work up to a fixed first, second, ... n-order, also when re-summed to ∞ order).
- AND contribution from collinear (COL) singularities, (from integration over small angles between photons and fast electrons) are large but finite;
  mass of electron acts as a natural regulator.

Collinear factorization in QCD: why?

Complications in the perturbative Quantum Chromodynamics (pQCD), in strong interactions at high energies $>>$ 1GeV:

- gluon and some quarks are *massless*, hence no convenient natural mass IR/COL regulators – it is the wave function of hadrons (confinement) which does the job,
- QCD has non-abelian charge *colour* instead of simpler electric charge of QED.

Theoretical and practical solutions to the above problems in pQCD has been found in early 80-ties, and today is referred to as "Collinear Factorization Theorems" of pQCD.
All present day practical pQCD calculation are based on them!

Basic toolbox of standard CFTs in pQCD

All present day practical pQCD calculation are based on the Collinear Factorization Theorems (CFTs).

CFTs provide the following basic toolbox and features:

- Definition of the PDF's = parton distribution functions; methodology of extracting them from data in one process, one energy scale and exploiting them in another process, another energy scale.
- definition of the hard process Matrix element (ME) and methodology of calculating it order by order in pQCD, (somewhat similarly as in QED).
- methodology of combining PDFs and hard process ME into predictions for semi-inclusive experimental observables in the collider experiments like total production and/or decay rates, effective mass and angle distributions, "fatness of the hadronic jets" etc.

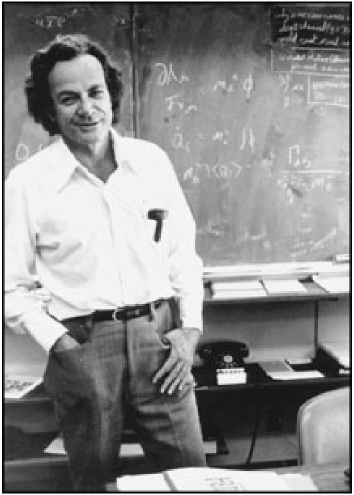
Shortcomings of the standard CFTs of the 80's

Standard CFTs of 80's are less and less suited for the present hadron collider experiments:

- They are ideal for total cross sections (decay rates) and one-dimensional inclusive distributions, with all other many particles integrated over, summed over.
- Present collider experiments can see final state in fine details and measure x-sections/rates factor $>$ 10 more precisely than 20 years ago, when standard CFTs were formulated,
- fast computers and powerfull stochastic simulation methods (Monte Carlo) allow nowdays to calculate not only total rates or 1-dim. inclusive distributions but also to simulate **EXCLUSIVE** multiparton final states! Like what is really seen in the modern collider experiments!
- CFTs of 80's not suited for present demands/opportunities! **CFTs are now a bottleneck in pQCD!!!**

OLD PARTON MODEL (prehistory of QCD)

Old parton model by lucid R. Feynman and clever J. Bjorken:

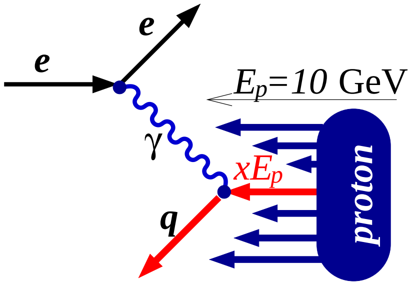


PDF=parton distr. function in Feynman parton model

Richard Feynman has stated:
quark q, as seen by the electron probe e, carries the same fraction x of the proton energy E_p, regardless how fast is proton:

$$\boxed{\frac{d\sigma}{dQ^2 dx} = \frac{d\sigma_{ep}}{dQ^2} D_{q\in p}(E_p, x), \quad D(E_p, x) = D(E'_p, x) = D(x)}$$

(proton energy $E_p = Q/x$ is in the Breit frame, see next slide)



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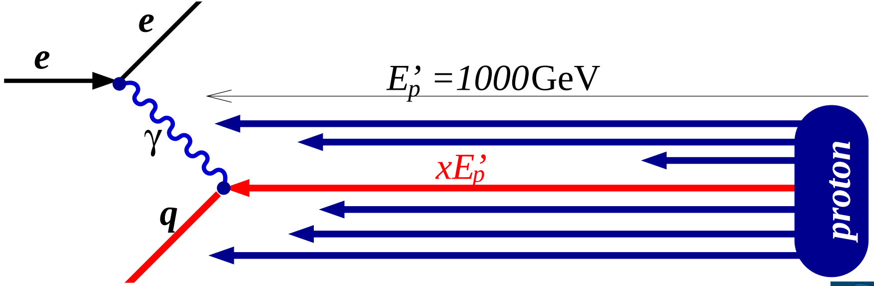
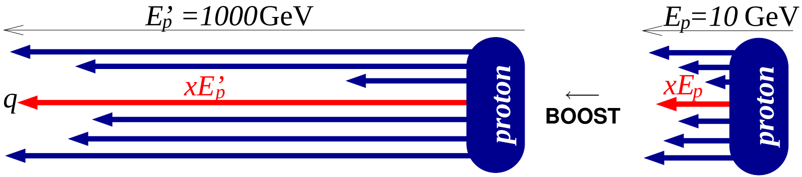


PDF=parton distr. function in Feynman parton model

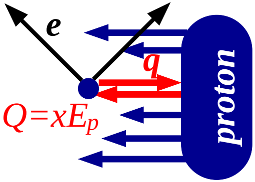
Feynman assumed that parton were non-interacting.
Then $\boxed{D(E,x) = D(E',x) = D(x)}$ for massles partons results directly and **TRIVIALLY** from a Lorentz z-boost!



$$p'_q = Bp_q = \begin{bmatrix} xE' \\ 0 \\ 0 \\ xE' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} xE \\ 0 \\ 0 \\ xE \end{bmatrix}, \quad E'_{proton} \equiv E'_p = \gamma E_p.$$

Deep inelastic scattering (DIS) in Breit frame

Strictly speaking Feynman's manipulation of PDF by means of slowing/accelerating proton works for DIS process in the Breit frame, where quark is backscatered and has energy $Q = \sqrt{Q^2}$, while proton energy is $E_p = Q/x$.



In reality Bjorken-Feynman conjecture in pQCD is broken, parton distribution (PDF) depends weakly on Q: $D(\ln Q, x)$. In QED this Q-dependence was discovered by Gribov and Lipatov; in pQCD it is attributed to Altarelli and Parisi.

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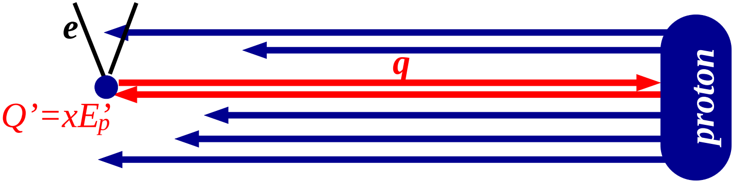

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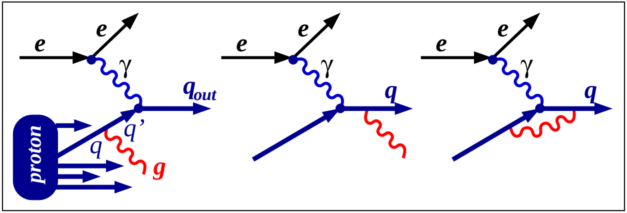
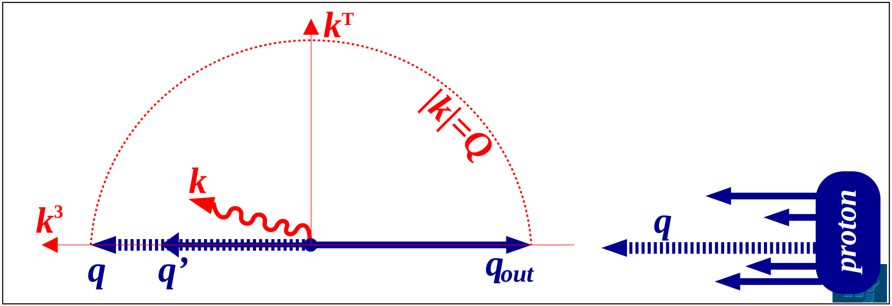
Why and where Feynman was "wrong"?

- Feynman indeed would be (almost) right in the small coupling limit $\alpha_S \to 0$,
  (what effectively happens in pQCD at extremely high energies)
- But in pQCD $\alpha_S \neq 0$ and let's find out the consequences.
- Assuming that we $\alpha_S << 1$, it is reasonable to start from Bjorken-Feynman scaling as a 0-th order approximation.
- To grasp the essence of the logarithmic violation of B-F scaling, it is enough to calculate 1-st order pQCD effect due to emission of a single real and virtual gluon, $1g$.
- The reason for the $\ln(Q)$ dependence of PDF will be that: $1g$ **correction** of pQCD to PDF changes by $\alpha_S \ln(Q'/Q)$, when comparing Q=10GeV and Q'=1000GeV DIS experiments. Let's look into this...

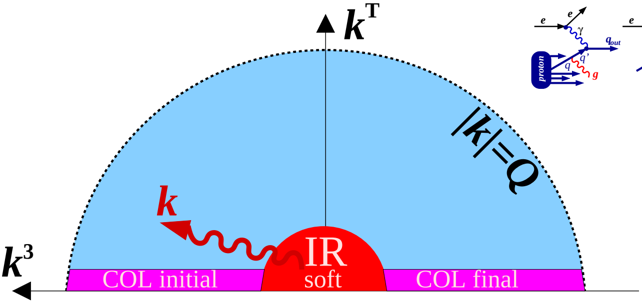
Emission of 1 gluon, 3 diagrams and phase space

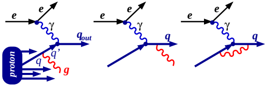


3 Feynman diagrams and the **real gluon** phase space in Breit frame:



Collinear and soft infrared (IR) singularities



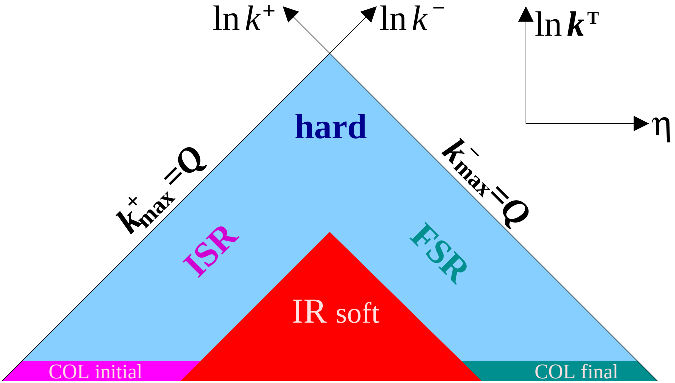


Gluon production ME (squared) has very strong peaks in the collinear and soft regions of the phase space (Breit frame Cartesian):

- IR soft: gluons with small energy $k^0 \to 0$
- COLLinear initial: gluon almost collinear with init. quark
- COLLinear final: gluon almost collinear with outgoing quark

Contributions from these IR regions are balanced by virtual contributions.

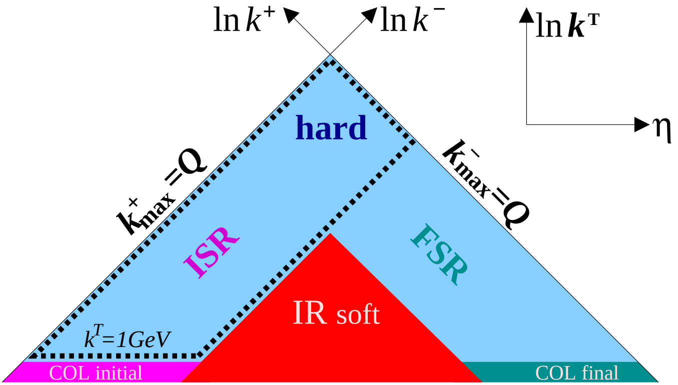
The same 1-gluon space in logarithmic variables



Three regions of general interest ($k^{\pm} = k^0 \pm k^3$):

- HARD: gluon in hard process
- ISR: initial state radiation
- FSR: final state radiation

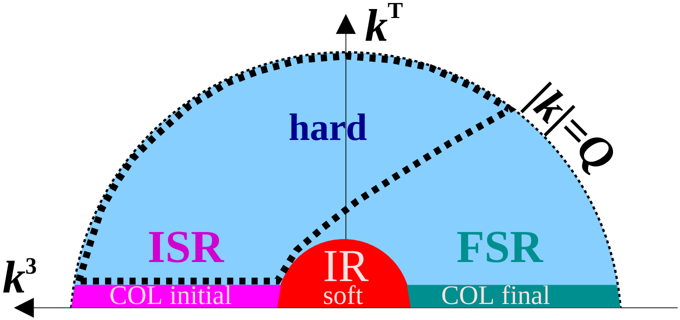
1-gluon space in logarithmic lightcone variables



Gluons from only two regions only can affect x_{Bj} and PDF(x_{Bj}):

- HARD: gluon in hard process
- ISR: initial state radiation
- **Interesting emission region inside dashed line.**

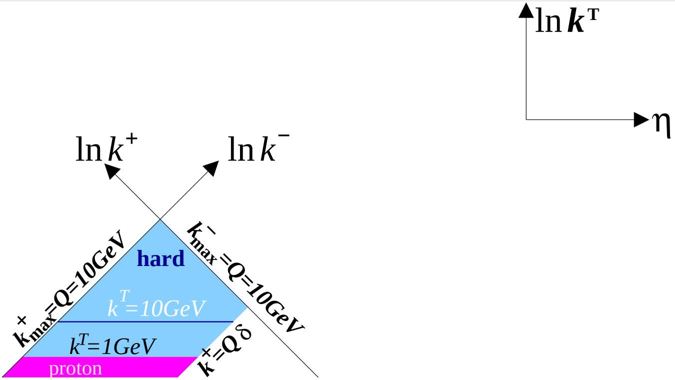
NB. The same ISR+HARD region in the Cartesian (non-log) variables looks as follows:



Let's concentrate on ISR+HARD,
unresolved FSR combined with virtuals.

ISR collinear divergence (both real and virtual) is regulated/cut-off by incalculable proton bound state wave function.

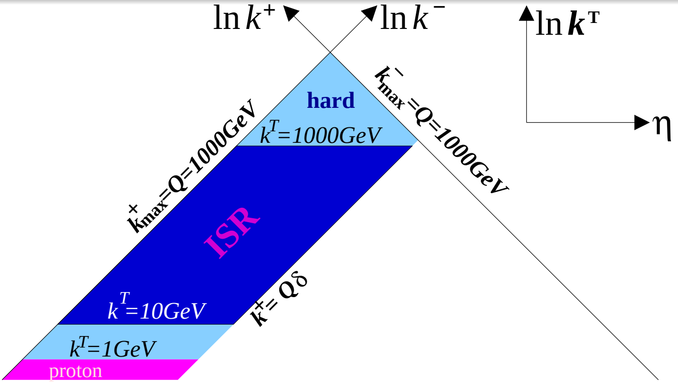
The source of ln(Q) dependence of PDF in a nutshell



What happens when we calculate 1-gluon correction to PDF twice, for $Q = 10$GeV and $Q' = 1000$GeV, and compare results? **In a nutshell:**

- the essential difference comes from the integral over the dark blue trapezoid segment of the length = $\ln(Q'/Q)$.
- IR soft and FSR gluons don't play any role!

The source of ln(Q) dependence of PDF in a nutshell



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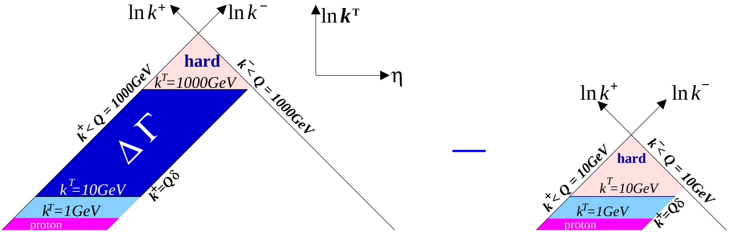
The same in terms of simple formulas



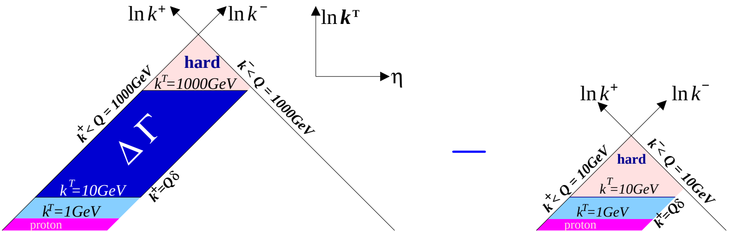
$$\frac{d\sigma}{dQ^2 dx} = \frac{d\sigma_{eq}}{dQ^2}\left[F^{(0)}(x) + F^{[1]}(Q,x)\right],$$

$$F^{[1]}(Q,x) = \frac{2C_F\alpha_S}{\pi}\int_{Q\delta}^{Q}\frac{dk^+}{k^+}\int_{m_p}^{Q}\frac{dk^-}{k^-}\,W(k^+,k^-,Q)F^{(0)}\Big(x+\frac{k^+}{Q}\Big) + F^{(0)}(x)V_{\rm virt.},$$

$$\Delta\Gamma\left(\frac{Q'}{Q}\right) = F^{[1]}(Q',x) - F^{[1]}(Q,x) = \frac{2C_F\alpha_S}{\pi}\int_{Q\delta}^{Q}\frac{dk^+}{k^+}\int_{Q}^{Q'}\frac{dk^-}{k^-}\,\omega\left(\frac{k^+}{Q}\right)F^{(0)}\Big(x+\frac{k^+}{Q}\Big)$$

The area of the dark blue segment= $\int_{Q\delta}^{Q}\frac{dk^+}{k^+}\int_{Q}^{Q'}\frac{dk^-}{k^-} = \ln\frac{1}{\delta}\,\ln\frac{Q'}{Q}$.

... and equivalent familiar formulas



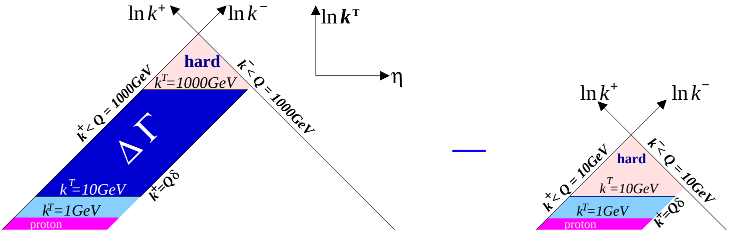
$$\Delta\Gamma\left(\frac{Q'}{Q}\right) = F^{[1]}(Q',x) - F^{[1]}(Q,x)$$

$$= \frac{2C_F\alpha_S}{\pi}\int_\delta^{1-x}\frac{d\alpha}{\alpha}\;\ln\frac{Q'}{Q}\;\frac{1+(1-\alpha)^2}{2(1-\alpha)}\;F^{(0)}\left(\frac{x}{1-\alpha}\right)$$

$$= \ln\frac{Q'}{Q}\int_x^{1-\delta}\frac{dz}{z}\left\{\frac{2C_F\alpha_S}{\pi}\frac{1+z^2}{2(1-z)}\right\}F^{(0)}\left(\frac{x}{z}\right), \quad \alpha = \frac{k^+}{k^+_{\max}}.$$

The part {...} is familiar Altarelli-Parisi QCD LO evolution kernel.

Hard (pink) and proton (blue+magenta) segments cancel! Why?



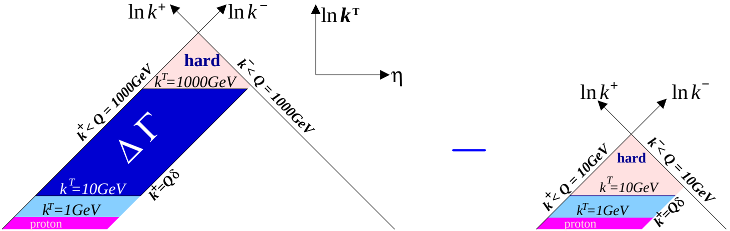
Proton (blue+magenta) segments cancel! Why?

The low k^T (light blue) proton segment (including corresponding virtuals):

$$F^{[1]}_{\rm pr.}(Q,x) = \frac{2C_F\alpha_S}{\pi}\int_{Q\delta}^{Q}\frac{dk^+}{k^+}\int_{m_p}^{Q}\frac{dk^-}{k^-}\; W(k^+,k^-,Q)F^{(0)}(x+\frac{k^+}{Q})+F^{(0)}(x)V_{\rm pr.},$$

and the fact that it is the same $F^{[1]}_{\rm pr.}(Q,x) = F^{[1]}_{\rm pr.}(Q',x)$,
is a highly plausible conjecture, which cannot be proved in pQCD,
because proton bound state wave function is involved.
However, we shall give precise prescription how to extract it from the data.

More discussion...



The hard emission (pink) segments cancel! Why?

For the high k^T (pink) hard segment (including virt.):

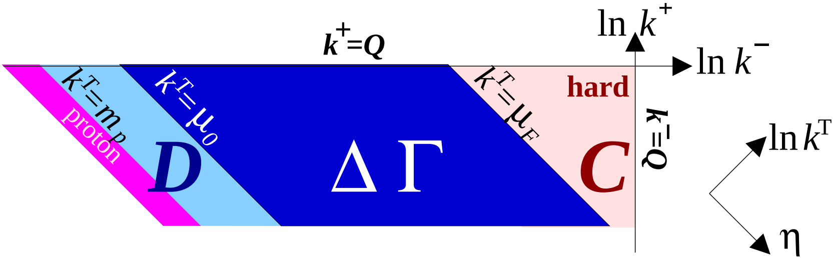
$$F^{[1]}_{\rm hard}(Q,x) = \frac{2C_F\alpha_S}{\pi}\int_{Q\delta}^{Q}\frac{dk^+}{k^+}\int_{\mu_F}^{Q}\frac{dk^-}{k^-}\; W(k^+,k^-,Q)F^{(0)}(x+\frac{k^+}{Q})+F^{(0)}(x)V_{\rm pr.},$$

the equality $F^{[1]}_{\rm hard}(Q,x) = F^{[1]}_{\rm hard}(Q',x)$ is provable in pQCD.

It results from the scale-invariance of the phase space and matrix element $W(k^+,k^-,Q) = W\left(\frac{k^+}{Q},\frac{k^-}{Q}\right)$.

The main issue is how to remove IR/Coll. singularities from it, see next...

For many gluons the same triple-factor structure



3 factors come from 3 segments of the phase space:

$$\frac{d\sigma}{dQ^2 dx} = \frac{d\sigma_{eq}}{dQ^2} F(Q,x),$$

$$F(Q,x) = \int_0^1 dz\, D\Big(\frac{\mu_0}{m_p}, x_0\Big) \int_0^1 dz\, \Delta\Gamma\Big(\frac{\mu_F}{\mu_0}, z\Big) \int_0^1 dz' C\Big(\frac{Q}{\mu_F}, z'\Big) \delta(x - z' z x_0)$$

[Struc.Funct.] = [Initial PDF] x [Evolution Segm.] x [Coefficient Funct.]

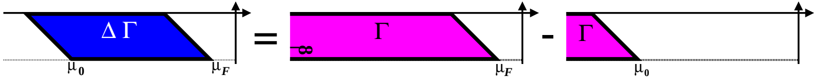
Three essential How to's of the Collinear Factorization toolbox

1. How to get “Evolution segment” ΔΓ from pQCD?

Back to 1-gluon case. Using pQCD we define/calculate:

$$\Delta\Gamma\left(\frac{\mu_F}{\mu_0}, z\right) = \Gamma(\mu_F, z, \varepsilon) - \Gamma(\mu_0, z, \varepsilon).$$

In a graphical form the above is:

$\Delta\Gamma =$ 

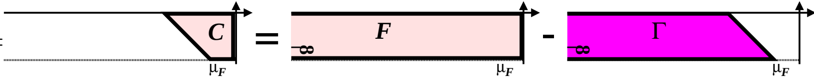
No proton in Γ any more! Pure pQCD! Massles quark!
Any kind of IR/Coll. regulator is OK in Γ, as it cancells in ΔΓ. For instance dimensional regularization factor $(k^T)^\varepsilon$ can be used, or massive quark and gluon, whatever.

Idealized “Bare PDF” $\Gamma(\mu_F, z, \varepsilon)$ is process-independent and divergent.

2. How do we get hard part C from pQCD?

In a graphical form the subtracted hard process ME ("Coefficient Function" in the "DIS slang") is defined as:

$C_{\mathrm{DIS}} =$



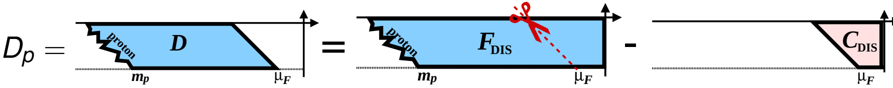
It is calculated in **pure pQCD** as an IR-finite difference:

$$C_{\mathrm{DIS}}(Q/\mu_F) = F_{\mathrm{bare}}(Q,\varepsilon) - \Gamma(\mu_F,\varepsilon).$$

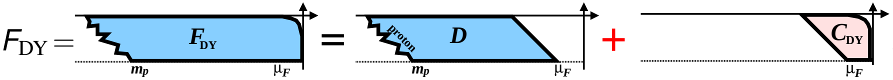
Finite C-function is **process dependent**!
It will be different in W/Z production and in DIS process.

3. How do we define, extract from data and exploit the process independent PDF of a hadron?

PDF is obtained by **from DIS exper. data** by means of "decapitating" the DIS Struc. funct. using $C_{\rm DIS}$ of pQCD:



Next, we may plug in this (process indep.) universal PDF, D_p, into a **prediction** for the DY process:

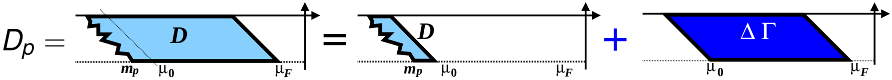


provided we have calculated/subtracted hard process ME for the DY process in pQCD.

("Beheaded Str.Fun."=PDF now gets new head:-)

Evolution of PDF and SF is the same, but...

The universal PDF evolves with the same evolution factor as the structure function, see graphic representation of that:

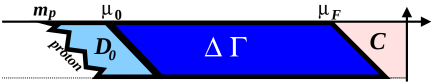


Although PDF is process-independent,
it is not only hadron-type-dependent,
but also factorization-scheme-dependent!

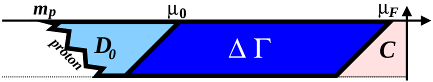
Last not least: Factorization Scheme (FACS) dependence

Observables, Structure Funct. are FACS-independent,
but 3 components, D_0, $\Delta\Gamma$ and $C_{\rm hard}$ are FACS-dependent.
Changing the "evolution variable" type generates FACS-dependence.

- Dimensional regularization, $\overline{MS}$, $t = \ln k^T$, k^T-ordering:



- Angular-ordering, $t = \eta = \ln(k^+/k^-)$, friendly to eikonal/soft limit:



- "DIS scheme", $t = \ln(k^-)$, $t = \ln Q$, virtuality-ordering:

For example, each of the above has a different size or absent numerically dominant component $\left(\frac{\ln(1-z)}{1-z}\right)_+$ in C.
[Altarelli-Ellis-Martinelli, Nucl.Phys B157 (1979)].

Summary on Factorization Theorems toolbox in pQCD

- pQCD can predict SF at Q, provided we measure it at Q_0,
- using clever construction of an universal PDF, we are able to “teleport” an experimental knowledge on partons inside fast hadron, from one process to another.
- hard part ME can be calculated order by order using Feynman diagrams (almost like in QED).
  We know how to combine it with PDFs.
- Can we use classic CFTs to construct Monte Carlo?
  Unfortunately, we cannot... See next part II.

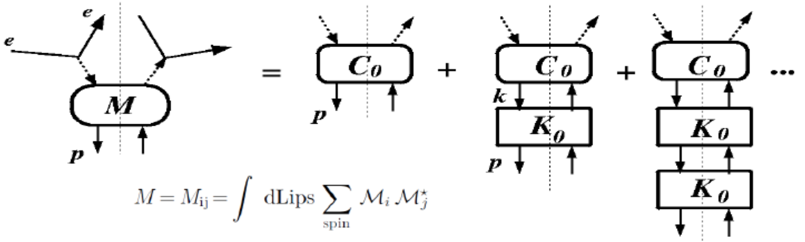
PART II
more on theory of the collinear factorization in a form suitable for the Monte Carlo

Part II: CFTs and Stochastic simulation (Monte Carlo)

- Classic CFTs were formulated by:
  △ 1979, EGMPR [Ellis, Georgi, Machacek, Politzer, Ross],
  △ 1980, CFP [Curci, Furmanski, Petronzio],
  △ 1980-85, CSS [Collins, Sterman , Soper] and others.
- Why not suited for the parton shower MC?
  ⋆ Over-subtractions
  ⋆ Non-conservation of the 4-momenta
- How to modify (cure) them?
  • Introduce time-ordered exponential at the early stage, already when isolating/subtracting coll/IR singular parts.
  • Redefine projection operators for extracting singular (coll.) parts such that 4-mom. is conserved.

EGMPR scheme of collinear factorization (1978)

"Raw" factorization of the IR collinear singularities



$$M = M_{ij} = \int \mathrm{dLips} \sum_{\mathrm{spin}} \mathcal{M}_i \, \mathcal{M}_j^*$$

- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0(1 + K_0 + K_0^2 + \cdots) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

Curci-Furmanski-Petronzio collinear factorization scheme (1979)

Curci Furmanski and Petronzio (CFP) have customized (1979-80) EGMPR to $\overline{MS}$ and exploited it in practice to NLO level.

$$F = C_0 \cdot \frac{1}{1-K_0} = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right),$$

$$= \left\{ C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0} \right\} \otimes \left\{ \frac{1}{1-\left(\mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)} \right\}_{\otimes},$$

$$\Gamma\left(\alpha, \frac{1}{\epsilon}\right) \equiv \left(\frac{1}{1-K}\right)_{\otimes} = 1 + K + K\otimes K + K \otimes K \otimes K + \dots,$$

$$K = \mathbb{P} K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}, \quad C = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}.$$

Ladder part Γ corresponds to MC parton shower
C is the hard process part
$\mathbb{P}$ is the projection operator: $\mathbb{P} = P_{spin}\, P_{kin}\, PP$

Over-subtraction problem in translating CFP/EGMPR into Monte Carlo

By examining to LO in CFP (real emissions)

$$F = C_0 \cdot \frac{1}{1-K_0} = C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0} \otimes \Gamma, \quad \Gamma = \frac{1}{1-\left(\mathbb{P}K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\right)},$$

we see enormous oversubtractions/cancellations:

$$\Gamma \simeq \frac{1}{1-\left(1-e^{-\frac{1}{\varepsilon}}\right)} = 1 + \left(1-e^{-\frac{1}{\varepsilon}}\right) + \left(1-e^{-\frac{1}{\varepsilon}}\right)^2 + \ldots$$

while from RGE and explicit LO calculations we get

$$\Gamma = e^{+\frac{1}{\varepsilon}} = 1 + \frac{1}{\varepsilon} + \frac{1}{2!}\frac{1}{\varepsilon^2} + \ldots$$

The same in EGMPR, translating ε-poles $\frac{1}{\varepsilon} = \int_0^{\mu_F} \frac{dk^T}{k^T} \left(\frac{k^T}{\mu_F}\right)^\varepsilon$ of CFP.

NO WAY to build Monte Carlo on that!

We need the exponent directly from the Feynman diagrams!!!

Correcting for over-subtractions

$$\boxed{F = \frac{1}{1-K_0} = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0] \cdot \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^{s}K_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu)}$$

$$\overleftarrow{\mathbb{R}}_\mu(K_0) = \overleftarrow{\mathbb{B}}_\mu\left[\frac{1}{1-K_0}\right] \equiv 1 + \overleftarrow{\mathbb{B}}_\mu[K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] + \overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] + \dots$$

No over-subtraction due to presence of:

- $\exp_{TO}$=time ordered exponential in the evolution variable = log of the factorization scale, next slide.
- Operator $\overleftarrow{\mathbb{B}}$ is defined recursively (similarly as β-functions in Yennie-Frautschi-Suura 1961 subtraction scheme):

$$\overleftarrow{\mathbb{B}}_\mu[K_0] = K_0 - \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] = K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \mathbb{P}'_\mu\{{}^{s_2}K_0 \cdot \overleftarrow{\mathbb{B}}_{s_2}[K_0]\} - \overleftarrow{\mathbb{B}}_\mu[K_0] \cdot \mathbb{P}'_\mu\{K_0\},$$

$$\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0 \cdot K_0] = K_0 \cdot K_0 \cdot K_0 - \mathbb{P}'_\mu\{{}^{s_3}K_0\} \cdot \mathbb{P}'_{s_3}\{{}^{s_2}K_0\} \cdot \mathbb{P}'_{s_2}\{{}^{s_1}K_0\} - \dots$$

- Modified $\mathbb{P}'_\mu$ new projection operator is the key point!
- $\mathbb{P} \rightarrow \mathbb{P}'_\mu$ conserves four-momentum, contrary to EGMPR.

Modified projection operator $\overleftarrow{\mathbb{P}}'_\mu$

- $\overleftarrow{\mathbb{P}}'_\mu$ does spin projection as $\mathbb{P}$ of CFP,
- $\overleftarrow{\mathbb{P}}'_\mu(A)$ extracts singular part from integrand A (not from $\int A$!)
- where A is *at most single-log* coll. divergent!
- $\overleftarrow{\mathbb{P}}'_\mu$ acts on integrand, leaves out Lorentz inv. phase space, sets on-shell all (cut) real momenta towards hard process
- $\overleftarrow{\mathbb{P}}'_\mu$ sets upper limit μ on the phase space for all real (cut) partons towards the hadron using $\mu > s(k_1, .., k_n)$,
- examples of $s(k_1, .., k_n) = \max(k_i^T/\alpha_i)$, or $s = \max(k_i^T)$,

Bottom line:

This new factoriz. sch. cures both problems of EGMPR/CFP:

(i) over-subtraction and

(ii) 4-momentum non-conservation.

Solid basis for MC featuring pQCD beyond LO, both in the hard part and the ladder part is in place.

Hierarchy of scales in T.O. exponential

$$\exp_{TO}\left(\mathbb{P}'_{\mu}\{A\}\right)(\mu) = 1 + \mathbb{P}'_{\mu}\{A\} + \mathbb{P}'_{\mu}\{^{s_2}A\}\cdot\mathbb{P}'_{s_2}\{^{s_1}A\}$$
$$+ \mathbb{P}'_{\mu}\{^{s_3}A\}\cdot\mathbb{P}'_{s_3}\{^{s_2}A\}\cdot\mathbb{P}'_{s_2}\{^{s_1}A\} + \ldots$$

For $A = \int dLips(k_1, k_2, ..., k_n)\; f(k_1, ..., k_n)$, with k_i being on-shell emitted partons, notation $\{^{s_3}A\}$ defines $s_3 = a(a_1, ..., a_n) = \max(a_1, ..., a_n)$ and in

$$\mathbb{P}'_{\mu}\{^{s_3}A\}\cdot\mathbb{P}'_{s_3}\{^{s_2}A\}\cdot\mathbb{P}'_{s_2}\{^{s_1}A\}$$

the entire integrand multiplied by

$$\theta_{\mu>s_3>s_2>s_1}$$

insted of EGMPR/CFP common limit:

$$\theta_{\mu>s_3}\;\theta_{\mu>s_2}\;\theta_{\mu>s_1}$$

PDFs, evolution and kernels...

In $F(Q) = C(Q,\mu) \cdot D(\mu)$, hard process part is $C(Q,\mu) = C_0 \cdot \overleftarrow{\mathbb{R}}_\mu[K_0]$.
We also define exclusive PDF (ePDF) as the integrand in:

$$D(\mu) = \exp_{TO}\left(\overleftarrow{\mathbb{P}}'\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}\right)(\mu) = \exp_{TO}(K).$$

LO and NLO truncations of the exclusive evolution kernel K_μ are:

$$K_\mu^{LO} = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0\right\}, \quad \text{taken at } \mathcal{O}(\alpha^1),$$

$$K_\mu^{NLO} = \overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0 + K_0 \cdot (1 - \overleftarrow{\mathbb{P}}') \cdot K_0\right\}, \text{ truncated at } \mathcal{O}(\alpha^2).$$

The standard inclusive x-dependent $D(\mu, x)$ obeys by construction an ordinary evolution equation:

$$\boxed{\partial_\mu D(\mu, x) = \mathcal{P} \otimes D(\mu)(x)}$$

with the traditional DGLAP inclusive kernel being

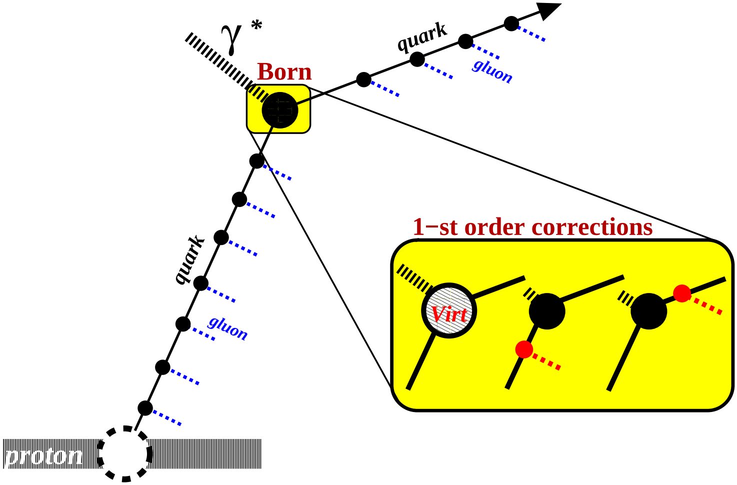
$$\mathcal{P}(x) = \int d\text{Lips}\; \delta\left(x = \frac{\sum k_i^+}{E_0}\right)\delta\left(1 - \frac{s}{\mu}\right)\overleftarrow{\mathbb{P}}'_\mu\left\{{}^sK_0 \cdot \overleftarrow{\mathbb{R}}_s[K_0]\right\}.$$

Another important ingredient:

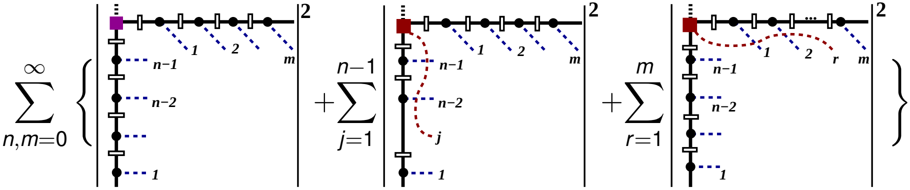
New method of inserting NLO corrections in the hard process and the ladder parts, similar to that of Yennie-Frautschi-Suura,

(which we have practiced in QED MCs for LEP)

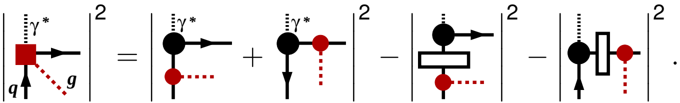
YFS-like method of NLO correcting HARD process



YFS-like method of NLO correcting HARD process

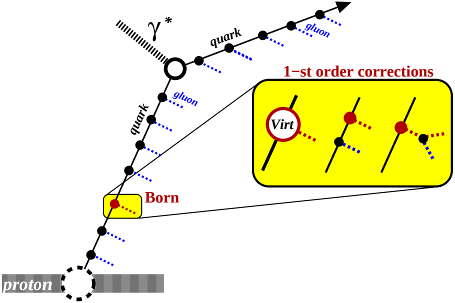


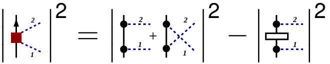
where the following NLO real/virtual corrections/distributions



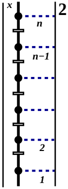
are free of any double or single collinear/soft singularities.

NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

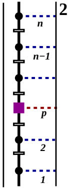




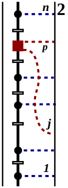
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Big\{$$



$$+ \sum_{p=1}^{n}$$



$$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$



$$\Big\} = e^{-S_{ISR}} \Big\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \Big(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \Big) \Big[\sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \Big] \delta_{x=\prod_{j=1}^{n} x_j} \Big\}.$$

Summary and Prospects

- New formulation of the collinear factorization, better suited for Monte Carlo implementation is proposed.
- NLO contributions to hard process and evolution kernels are already recalculated up to NLO in the new scheme (non-singlet NLO exclusive kernels calculated).
- Implementation in the Monte Carlo is tested at the prototype level; critical MC weights are examined numerically.
- More work to be done, towards new MC for W/Z production at LHC/Tevatron and DIS process.

1970 1980 1990 2000 2010

LO

Moments OPE — (74) QCD: Georgi+Politzer

Diagramatic — (72) QED: Gribov+Lipatov; (77) Altarelli+Parisi

Monte Carlo — 10 years → (85) Sjostrand; (88) Marchesini,Webber

NLO

Moments OPE — (78) Floratos+Ross+Sachrajda

Diagramatic — (81) Curci+Furmanski+Petronzio

Monte Carlo — (92) Kato et.al.; 27 years later → (08) Jadach Skrzypek

WE ARE HERE!!!

NNLO

Moments — (03) Moch+Verm.+Vogt

Diagramatic — (03) Moch+Verm.+Vogt

Monte Carlo — (15) ???