On regularizing the infrared singularities in QCD NLO splitting functions with the new Principal Value prescription

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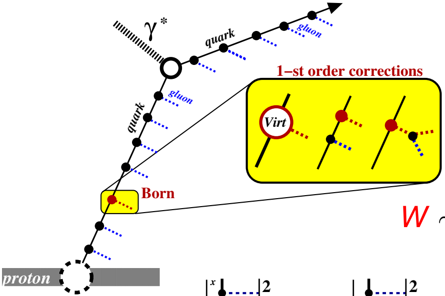
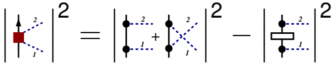
Plan

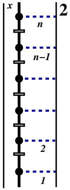
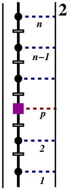
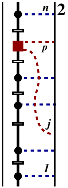
- Introduction
- New Principal Value prescription
- Results for NLO splitting functions
- Summary

NLO-corrected middle-of-the-ladder kernel, C_F^2

from S. Jadach talk



$W \sim$ 

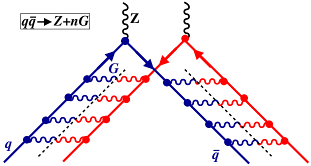
$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Big\{$  $+ \sum_{p=1}^{n}$  $+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$  $\Big\} = e^{-S_{ISR}} \Big\{ \delta_{x=1} +$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \, \rho_{1B}^{(1)}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Big\}.$$

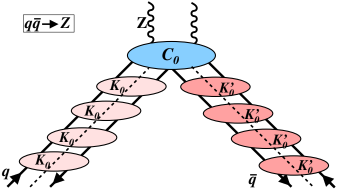
Construction of the evolution kernel

in collinear factorization

LO cascade and
construction of the ladder

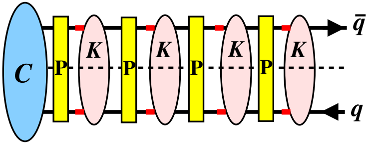


Include NLO and
group graphs in the ladder into
"two-particle-ireducible" sets K



Use "projection operators" P to
split the ladder and extract kernels

$$\Gamma_{qq} = \mathrm{Tr}\Big[\frac{\hat{n}}{4nq} K \hat{p}\Big]$$



PV prescription

☺ Axial gauge = physical interpretation as parton shower
☹ Axial gauge = spurious (unphysical) singularities

$$\text{gluon propagator:} \quad \frac{1}{l^2}\left(g^{\mu\nu} - \frac{l^\mu n^\nu + n^\mu l^\nu}{nl}\right)$$

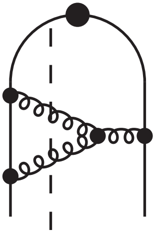
Spurious singularities cancel in full set of diags, but need regularization
Curci, Furmanski, Petronzio [80] Ellis, Vogelsang [96], Heinrich, Kunszt [97]:

$$\text{Principal Value:} \quad \left[\frac{1}{nl}\right]_{PV} = \frac{nl}{(nl)^2 + \delta^2 (pl)^2}$$

PV is more like "phenomenological rule"

Rigorous prescription: Mandelstam [83], Leibbrandt [84].
Difficult in calculations: Bassetto, Heinrich, Kunszt, Vogelsang [97].

Problem with real emission graph



Standard Heinrich, Kunszt [1998]:

$$N(\epsilon, Q^2)\left[\frac{P_{qq}(x)}{\epsilon^3} - 2I_0\frac{P_{qq}(x)}{\epsilon^2} + \frac{p_{qq}(x)}{\epsilon}\left(-2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\right)\right] + \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

Not good for Parton Shower, needs Real-Virt. cancel.

Parton Shower oriented Jadach, et.al. [2011]:

$$\frac{p_{qq}(x)}{\epsilon}\left(+2I_1 + 4I_0 + 2I_0\ln x - 2I_0\ln(1-x)\right) + \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

Good for PS, δ is a cut-off in 4-dimensions, easy to generate in MC:

$$I_0 = \int_0^1 \frac{dx}{[x]_{PV}} \sim \int_\delta^1 \frac{dx}{x} = -\ln\delta, \qquad I_1 = \int_0^1 \frac{dx\ln x}{[x]_{PV}} \sim -\frac{1}{2}\ln^2\delta,$$

$$P_{qq}(x) = p_{qq} + \epsilon(1-x), \qquad p_{qq} = \frac{1+x^2}{1-x}$$

New use of PV prescription

Standard: regularize with PV only the gluon propagator
leave other singularities in (+)-component of integration momenta

$$\frac{d^m l}{l_+^{1-\epsilon}}, \qquad l_+ = \frac{nl}{np}$$

New proposal: regularize with PV all singularities of the integrand
in (+)-component of integration momenta, real & virtual

$$\frac{d^m l}{l_+^{1-\epsilon}} \to d^m l \left[\frac{1}{l_+}\right]_{PV} \left(1 + \epsilon \ln l_+ + \epsilon^2 \frac{1}{2} \ln^2 l_+ + \dots\right)$$

All (+)-singularities cancel in the final expression (kernel), so extension of "phenomenological PV rule" of Curci-Furmanski-Petronzio possible

Example: virtual three point integral

Must perform (+)-integral as the last one, Ellis, Vogelsang [1996]:

$$\int \frac{d^m l}{(2\pi)^m} \; \frac{f(l_+)}{l^2(l-q)^2(l-p)^2} =$$

$$= \frac{-i}{16\pi^2 q^2}\left(\frac{4\pi}{-q^2}\right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{-\epsilon}\left[\int_0^x dy\, f(l_+) \frac{z^\epsilon(1-z)^\epsilon}{1-y}\left(1+2\epsilon\ln\frac{1-y}{1-z}\right)\right.$$

$$\left. + 2\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}(1-x)^{-\epsilon}\int_x^1 dy\, f(l_+)(1-y)^{-1+2\epsilon}\right],$$

$$x = q_+/p_+, \quad y = l_+/p_+, \quad z = y/x, \quad p^2 = (p-q)^2 = 0, \quad m = 4+2\epsilon,$$

Singularities at $y = 0$ and $y = x$: only from gluon propagator.

Singularity at $y = 1$: not from gluon propagator!

Proposal: treat all (+)-singularities on equal footing

Note: (+)-singularities lead to $1/\epsilon^3$ poles in kernel

Example: scalar non-axial integral

kinematics: $p^2 = (p-q)^2 = 0$

$$J_3^{\rm F} = \int \frac{d^m l}{(2\pi)^m} \frac{1}{l^2 (q-l)^2 (p-l)^2}$$

The PV prescription:

$$J_3^{\rm F} = C \left(-\frac{1}{\epsilon^2} + \frac{\pi^2}{6} \right), \qquad C = i \frac{\Gamma(1-\epsilon)}{(4\pi)^2 |q^2|} \left(\frac{4\pi}{|q^2|} \right)^{-\epsilon}$$

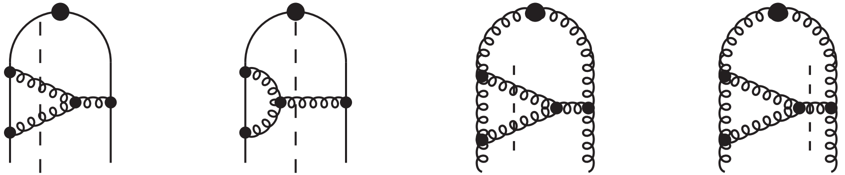
New PV prescription:

$$J_3^{\rm F} = C \left(-\frac{2 I_0 + \ln(1-x)}{\epsilon} - 4 I_1 + 2 I_0 \ln(1-x) + \frac{\ln^2(1-x)}{2} \right),$$

$\frac{1}{\epsilon^2}$ replaced by I_1 and $\frac{1}{\epsilon} I_0$

NLO kernels P_{qq} and P_{gg} in New PV scheme

There are only four graphs with $1/\epsilon^3$ singularity:

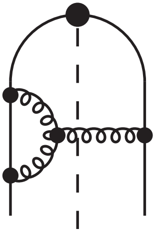


$\tilde{\Gamma}^{(d_R)}_{qq}(x,\epsilon)$ $\tilde{\Gamma}^{(d_V)}_{qq}(x,\epsilon)$ $\tilde{\Gamma}^{(d_R)}_{gg}(x,\epsilon)$ $\tilde{\Gamma}^{(d_V)}_{gg}(x,\epsilon)$

related to P_{qq} and P_{gg} splitting functions in a standard way:

$$\tilde{\Gamma}_{qq(gg)}(x,\epsilon) = \delta_{1-x} + \frac{1}{\epsilon}\left(\frac{\alpha_S}{2\pi}P^{LO}_{qq(gg)}(x) + \frac{1}{2}\left(\frac{\alpha_S}{2\pi}\right)^2 P^{NLO}_{qq(gg)}(x) + \ldots\right) + \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$$

Virtual contribution to NLO P_{qq} kernel in NPV



The virtual graph contributes:

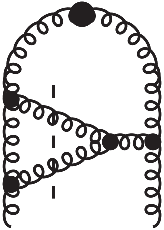
$$\tilde{\Gamma}_{qq}^{(d_V)}(x,\epsilon) = -\frac{1}{\epsilon^2} P_{qq}\big(1+\epsilon\ln(1-x)\big)\tilde{Z}_{d_V}$$

$$-\frac{1}{\epsilon} p_{qq}\Big[I_0\big(2\ln x + 2\ln(1-x)\big) - 6I_1 - \mathrm{Li}_2(1-x) + \ln^2 x - 3 + \frac{8}{12}\pi^2\Big] + \frac{1}{\epsilon}\frac{1}{2}\frac{1+x}{1-x},$$

$$\tilde{Z}_{d_V} = 4I_0 + 2\ln(1-x) + \ln x - \frac{3}{2},$$

Inclusive sum of Real and Virtual graphs $\tilde{\Gamma}_{qq}^{(d)}$ identical as in standard PV scheme

Real contribution to NLO P_{gg} kernel in NPV



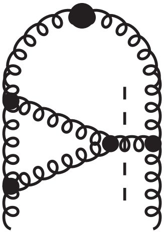
Only ϵ^{-1} poles
$\rightarrow$ the calculation can be done in 4-dimensions,
$\rightarrow$ much simpler than in standard PV

The real graph in New PV prescription:

$$\tilde{\Gamma}_{gg}^{(d_R)}(x,\epsilon) = C_S^{(d_R)} \frac{1}{\epsilon} \Big[P_{gg} \Big(-4 I_1 + 4 I_0 (\ln(1-x) - \ln(x) - 2) + 2\ln^2(1-x) + 2\ln^2(x) - 4\ln(x)\ln(1-x) - 8\ln(1-x) + \frac{11}{3}\ln(x) + 2\frac{\pi^2}{6} + 4 \Big) + \ln(x)\Big(\frac{11}{3}x^2 + \frac{23}{6}x + \frac{23}{6} + \frac{11}{3x}\Big) - \frac{22}{3}x^2 + \frac{24}{3}x - \frac{25}{3} + \frac{22}{3x} \Big].$$

Virtual contribution to NLO P_{gg} kernel in NPV

The virtual graph in New PV prescription:



$$\Gamma^{(d_V)}_{gg,NPV}(x,\epsilon) = C_S^{(d_V)} P_{gg} \Big[\frac{1}{\epsilon^2}(1+\epsilon\ln(1-x))\tilde{Z}^V_{GS} + \frac{1}{\epsilon}\Big(4I_0\ln(1-x) + 8I_0\ln(x) - 16I_1 + 4\ln^2(x) + 12\frac{\pi^2}{6} - \frac{134}{9}\Big)\Big] - C_S^{(d_V)}\frac{1}{3\epsilon}x$$

$$\tilde{Z}^V_{GS} = 12I_0 + 4\ln(1-x) + 4\ln(x) - \frac{22}{3},$$

Inclusive sum of Real and Virtual graphs $\tilde{\Gamma}^{(d)}_{gg}$ identical as in standard PV scheme

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Both schemes, PV and New PV, give the same P_{qq} and P_{gg} kernels

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Summary

- We modified the way of using the PV prescription in the light-cone gauge by applying it to all the singularities in the plus component of the integration momentum.
- In the New PV prescription the NLO kernels P_{qq} and P_{gg} are reproduced correctly.
- Partial results of four graphs contributing to the kernels, differ in PV and New PV: the $1/\epsilon^3$ poles, present in PV, are replaced by $(1/\epsilon)\ln^2\delta$ etc.
- Real graphs, free of $1/\epsilon^3$ and $1/\epsilon^2$ poles, can be calculated in four dimensions and are usable for the stochastic simmulations.
- The higher order poles cancel separately for real and for virtual components.